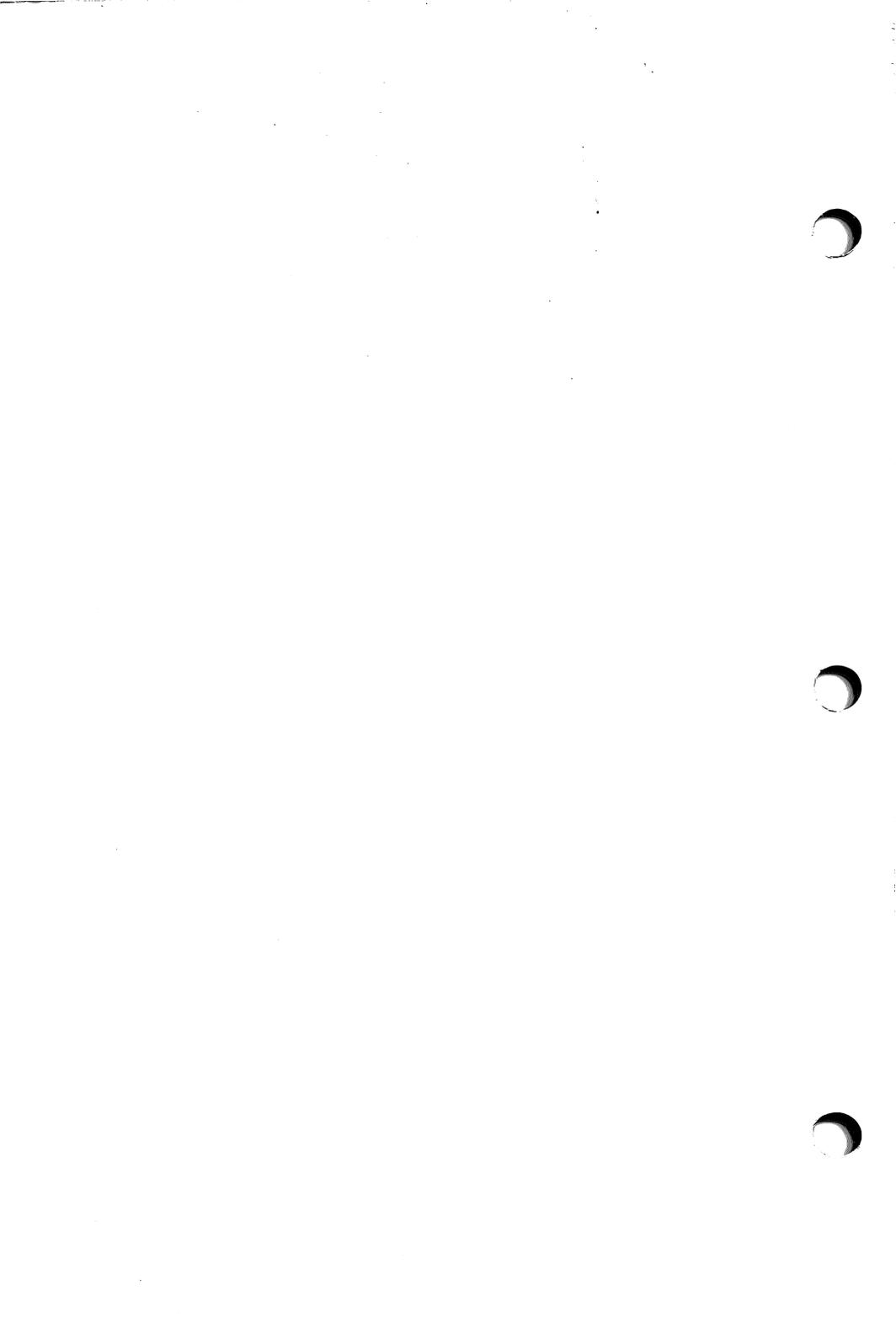




ARB PLOT

CONDUIT
Reviewed
and
Tested
Materials

**Instructor's
Guide**



ARBLOT

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Colorado School of Mines
Mark Harris,
Appalachian State University**

Instructor's Guide

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Iowa City, IA 52244 and Harper and Row Publishers, Inc., 10
East 53rd Street, New York, NY 10022.

Partial support for preparation of this material for
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Equipment: ARBPLOT requires an Apple II or Apple II Plus
computer with 48K, Applesoft ROM or Language System, and one
disk drive with DOS 3.3 (16 sector) controller.

Authors' Note

ARBPLOT is a set of computer graphics tools for use on the Apple II computer in teaching and learning calculus and related subjects. It is designed to work with any course organization and textbook. This manual is designed to help you master those tools.

Most of the manual is tutorial. You can best use it by sitting at your Apple II computer with your ARBPLOT diskettes, one or two blank diskettes, and this manual, and working your way through Chapters 2 and 3. Then sample the problems in Chapter 5, trying variations of your own.

Chapter 4 gives some advice on effective use based on our experience. Chapter 6 gives some technical details for those who wish to know how ARBPLOT works in order to write their own programs.

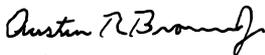
Our intent is to provide tools to illustrate any geometric concept or process in analytic geometry, differential calculus, and integral calculus of the plane. We would like to hear how you have used the programs and where they fall short.

The original design of what became ARBPLOT was begun by Brown in 1978 and was underwritten in part by a sabbatical from the Colorado School of Mines and NSF LOCI Grant No. SER-7900604 in the fall of 1979. Many revisions and expansions have occurred since then, the most significant being a complete redesign and rewrite as a result of CONDUIT reviewer suggestions in January 1982.

Harris's work in microcomputer graphics began in 1981 with several programs written for use in mathematics courses at Appalachian State University. The association with Brown followed a demonstration of these programs at a meeting of the North Carolina Educational Computing Service and a discussion with David Smith of Duke University, who had seen Brown's work through his role as a reviewer for CONDUIT.

Our thanks go to Hal Peters of CONDUIT and to David Smith who brought us together and have been most supportive of our work, to the CONDUIT reviewers for their advice, and to colleagues who have tried the various versions of the programs, particularly to Robert Fisk of the Colorado School of Mines. We are most grateful to the people of A.P.P.L.E., Neil Konzen and others, for their Program Line Editor and Program Global Editor, to the designers of word processing for the Apple (Applewriter and Magic Window), and to Christopher Espinosa for his high resolution character generator. This project would have been impossible without them.

Finally, and most importantly, our thanks go to our wives for understanding support through long months of being computer widows while we spent days and nights sweating over a hot computer.



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1. Introduction

ARBPLOT is a collection of three groups of computer graphics programs for the Apple II computer which can help you teach calculus and other mathematics subjects. The first group plots curves from functions defined in several ways; the second illustrates concepts in calculus; and the third creates and runs "slide shows," an aid to presenting preplanned sequences of graphic illustrations. The programs are on two diskettes.

This manual first describes each program and shows how to use the alternatives within the program. The manual then presents tactics for using the programs in various classroom situations, including planning and showing slide shows. Next, thirty classroom applications are presented, and finally, technical notes are given for the reader who wants a deeper understanding of the working of the programs.

The entire package is menu driven. You need not understand programming in order to use it. However, you must be able to describe functions in the way that Applesoft BASIC requires; this is described in Section 2.4.

1.1. Scope

The following outline presents the major topics and capabilities of the ARBPLOT programs, with references to the diskettes on which the programs are found.

- A. Function Plotting (disk 1, choices 1-4).
 - 1. General. Properties of all programs.
 - a. Functions.
 - (1) Continuous or discontinuous.
 - (2) Bounded or unbounded.
 - (3) Domain continuous or with gaps.
 - b. Scaling.
 - (1) Automatic (fill screen).
 - (2) Manual (windowing).
 - c. Display.
 - (1) Clear previous display.
 - (2) Overlay new display on old.
 - d. Curves.
 - (1) Solid line.
 - (2) Dotted line.
 - e. Axes.
 - (1) Show axes.
 - (2) Do not show axes.
 - 2. Coordinate systems.
 - a. Rectangular - explicit (disk 1, choice 1).
 - b. Rectangular - parametric (disk 1, choice 2).
 - c. Polar - explicit (disk 1, choice 3).
 - 3. Plot two curves simultaneously (disk 1, choice 4).
 - a. Rectangular - explicit only.
 - b. Plot a function with its inverse.
 - c. Flag the zeros of the second curve.
- B. Analytic Geometry - Conic Sections (disk 2, choice 1).
 - 1. Sum and difference definitions.
 - 2. Eccentricity definitions.
 - 3. Translation of axes.
 - 4. Rotation of axes (disk 2, choice 2).

- C. Limits and Derivatives (disk 1, choice 5).
1. Epsilon-delta definition of the limit of a function.
 2. Tangent as limit of secants.
 3. Mean Value Theorem for derivatives.
 4. Derivative as function (disk 1, choice 6).
- D. Integration.
1. General.
 - a. Antiderivative - include or omit.
 - b. Number of subdivisions - variable.
 - c. Difference of functions or single function.
 - d. Function must be defined throughout the interval.
 2. Definite integral in rectangular coordinates (disk 2, choice 3).
 - a. Rectangles (Fundamental Theorem).
 - b. Trapezoids.
 - c. Parabolas (Simpson's Rule).
 - d. All three simultaneously.
 - e. Show typical element.
 - f. Richardson extrapolation (after trapezoidal).
 3. Area - Rectangular coordinates - Monte Carlo (disk 2, choice 5).
 4. Area - Polar coordinates (disk 2, choice 6).
 5. Arc length (disk 2, choice 7).
 - a. Rectangular - explicit.
 - b. Rectangular - parametric.
 - c. Polar.
 6. Indefinite integral (disk 2, choice 4).

- E. Roots of an Equation (disk 1, choice 7).
 - 1. Bisection Method.
 - 2. Regula Falsi.
 - 3. Newton's Method.
- F. Sequences and Series (disk 1, choice 8).
 - 1. Elements of a sequence.
 - 2. Partial sums.
 - 3. A function of the elements or a second sequence.
- G. Differential Equations - First Order (disk 2, choice 8).
 - 1. Direction field.
 - 2. Integral curves.

2. Using ARBPLOT

2.1. Getting Started*

Begin by booting the system. If this expression is unfamiliar to you, insert the ARBPLOT diskette into the disk drive (Drive #1, if there are two drives), close the door, and turn the power on (switch at left rear of the Apple). On most Apples, the red light on the disk drive will come on, the diskette will spin, and the ARBPLOT main menu will appear. If the red light does not come on, look at the display screen. If you see "J" followed by the blinking cursor, type "PR#6", press "RETURN", and the booting process should begin. If you see "*" followed by the blinking cursor, press the "RESET" key, type "6" and then Ctrl-K (hold down the "CTRL" key while pressing "K"), press "RETURN", and the booting process should begin. If none of these work, ask your local Apple expert or your dealer what you did wrong or what is wrong with the computer. (Are the Apple and the TV plugged in? Is the TV connected to the Apple?)

*Note on Backup: The first step in using ARBPLOT is to make working copies of the two diskettes. ARBPLOT uses a special arrangement of data on the disks to speed up loading time. Standard copy procedures will destroy that arrangement, slowing the running of the programs. The original diskettes furnished with this package have the normal arrangement and load times. Follow the instructions in Appendix H to make working copies which load programs faster than normal.

Once you have booted the system, an introductory message will briefly appear, then the ARBPLOT main menu will be displayed on the screen. If you booted Disk 1, you will see:

```
><><><>ARBPLOT<>VERSION2.1<>DISK1<><><<
<
> 1. PLOT Y(X) <
< <
> 2. PLOT X(T) VS. Y(T) <
< <
> 3. PLOT R(T) (POLAR) <
< <
> 4. PLOT Y1(X),Y2(X), SIMULTANEOUSLY <
< <
> 5. LIMITS & DERIVATIVES <
< <
> 6. DERIVATIVE FUNCTION WITH EXTREMA <
< <
> 7. ROOTS OF F(X) = 0 <
< <
> 8. SEQUENCES & SERIES <
< <
> 9. SAVE DISPLAY <
< <
> 0. EXIT <
< <
><COPYRIGHT 1982><>*<SELECT (0-9)?<><><<
```

If you booted Disk 2, you will see:

```
><><><>ARBPLOT<>VERSION2.1<>DISK2<><><<
<
> 1. CONIC SECTIONS <
< <
> 2. ROTATION OF AXES; ELLIPSE <
< <
> 3. DEFINITE INTEGRAL <
< <
> 4. INDEFINITE INTEGRAL <
< <
> 5. MONTE CARLO INTEGRATION <
< <
> 6. POLAR INTEGRATION <
< <
> 7. ARC LENGTH <
< <
> 8. DIFF. EQUATIONS/DIRECTION FIELDS <
< <
> 9. SAVE DISPLAY <
< <
> 0. EXIT <
< <
><COPYRIGHT 1982><>*<SELECT (0-9)?<><><<
```

You are now ready to begin using ARBPLOT.

2.2. Choices

Each choice on the menu runs one of the ARBPLOT programs. Choices 1-8 on each diskette run the programs outlined above. Choice 9 saves the display for a "slide show" or printed copy. Choice 0 permits you to load the other ARBPLOT diskette or exit to Applesoft BASIC. To make a choice, press the appropriate key (0-9); you need not press "RETURN".

You will automatically return to the main menu when you elect to end a program. If you have exited to Applesoft, type "RUN APMENU" to return to the menu.

2.3. An Example: Plotting a Curve

Now let's see how a typical ARBPLOT program works. We shall plot the function $y=x(x-1)^{2/5}$ from $x=-1$ to $x=2$. Boot the system with Disk 1. Choose #1, PLOT Y(X). (Note: "1" is the key in the upper left corner of the keyboard, not the letter "L".) The red light will come on, the disk will spin, the program ARBPLOT1 will be loaded, an introductory message will briefly appear, and the following menu will be displayed:

>>ARBPLOT1 - CURRENT VALUES & CHOICES<<

1. FUNCTION

$$Y(X)=X*((X-1)^2)^.2$$

2. LIMITS/SCALING DELTA X= .05
 XMIN=-1 XMAX=2
 YMIN=0 YMAX=0
 SCALING: AUTOMATIC

3. SCREEN: CLEAR PRESENT DISPLAY

4. CURVE: SOLID LINE

5. AXES: SHOW

ACTIONS:

6. SHOW CURRENT DISPLAY

7. PLOT CURVE

8. EXIT

CHOICE: CHANGE VALUE(1-5)

 ACT(6-8)

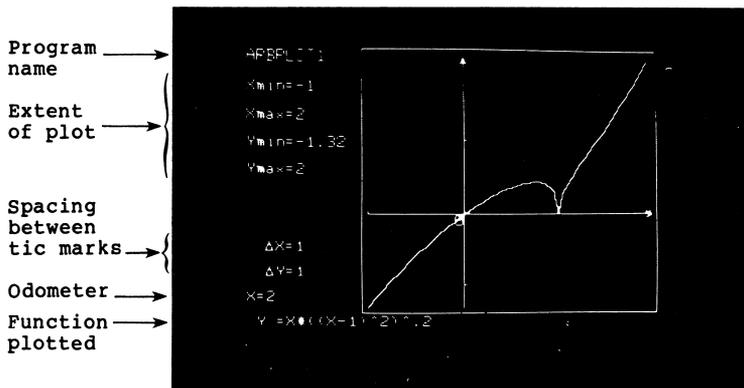
 WHICH(1-8)?

This menu is typical of the ARBPLOT program menus. Let's examine it in detail. The top line always gives the name of the program, followed by current values for up to eight choices, each of which may be selected by pressing the appropriate key (1-8). You need not press "RETURN". In most cases single key responses in an ARBPLOT program need not be followed by "RETURN". However, if you press the key for a response and nothing happens, press "RETURN". In the majority of places where you are to enter a numerical value, simply pressing "RETURN" (not preceded by any number) will retain the current value.

The choices are grouped in two sets; the first set changes values used by the program, and the second set takes actions. Let's first examine the actions, then return to the choices of values.

Press "6". You see a blank screen, with "End" in the lower right hand corner. You are viewing the current display; it is blank since you haven't yet created anything. Choice 6 on any of the program menus shows what is currently on the graphics display screen, whether prepared by the program you are presently running or some earlier program. To return to the menu, press any key.

Now press "7". "-- CALCULATING --" appears in the middle of the screen for about 8 seconds (how long depends upon the function and number of points plotted), then the following graph is drawn:



Displays are drawn and labelled in similar fashion for each program. First, the program name and the border are drawn. If you are showing the axes (choice 5 on the menu), and the axes pass through the plotting area, the axes will be drawn with tic-marks showing the scale. ΔX and ΔY on the left edge of the display show the spacing between tic-marks. If the axes are to be shown but do not pass through the plotting area, the tic-marks appear on the border of the plot. Finally, before

plotting, the equation of the function is printed below the plot area.

As the curve is plotted, an odometer runs on the left of the screen, showing the instantaneous value of the independent variable. You may interrupt the plotting by pressing any key; pressing a key again continues the plot. These two features, odometer and interrupt, occur only in the plot programs; they do not appear while the curve is being drawn in the other programs. Interrupt is possible during further processing in some programs. If you wish to stop plotting and return to the program menu before completion, press Ctrl-E after you have interrupted plotting. With some programs you will return instantly; with others, you must press a key after "End" appears.

When the plot is complete, the minimum and maximum values of the variables appear to the left of the plot. If automatic scaling (choice 2) has been selected, the values are the extent of the plot area. If manual scaling has been selected, the values are the calculated limits, not necessarily the extent of the area. However, minimum and maximum values in the menu are the extent of the area on the screen.

Finally, the word "End" appears in the lower right hand corner. Press any key to return to the menu. From this menu page you can press "6" to show the display again or you can leave the current program and return to the main ARBPLOT menu by pressing "8" -- but don't do this yet.

Now let's look at choices 1-5 on the ARBPLOT1 menu.

Choice 1 allows you to change the function. Defining and changing the function are described in the next section (2.4).

Choice 2 sets the limits of the plot. You must first set minimum and maximum values for the independent variable, X, then the increment between points to be plotted. Change the limits to 0 and 1 with increment .05, pressing "RETURN" after each number you enter. If you wish to retain the previous value of a particular number, just press "RETURN". If you make a mistake as you are entering a number, you may use the left arrow (←) key to backspace and correct before you press "RETURN". Note that you are asked whether the current choice is okay. Respond with "Y" or "N"; you need not press "RETURN".

The vertical (Y) limits may be specified in two ways. In automatic scaling, all of the points to be plotted are calculated in advance, and the minimum and maximum calculated values are taken as the bottom and top of the plot area. In manual scaling, you set the vertical limits of the plot area. If the plot goes off-scale, it will be clipped. Now set scaling to manual, the minimum value of Y to $-.4$, the maximum to $.4$, and plot the curve (choice 7). Note how the odometer keeps moving even when apparently nothing is being plotted. Note also there was no "- - CALCULATING - -" before plotting; all calculations occur while plotting.

Now return to the menu. Choices 3, 4, and 5 control conditions of plotting. With choice 3 you may overlay one plot on another. With choice 4 you may plot a dotted line. With choice 5 you may suppress the axes. Each of these is a two-way choice; pressing the key makes the change. Now press "4", then "5", then "7". The curve appears as a dotted line with no axes

showing. Return to the menu, and press "3", "4", and "7". The curve appears as a solid line overlaid on the previous dotted line. In programs other than plot programs, choices 3-5 control the conditions peculiar to the program. This example is typical of the procedures you follow for any of the programs; the actions and features particular to each program are described in Chapter 3.

ARB PLOT is intended to be free of errors and to protect you from any mistakes you might make. See Appendix E for a discussion of what to do if an error occurs despite all of our efforts.

2.4. Defining the Function

Choice 1 in each program permits you to change the function to be plotted. Some programs permit or require two functions; some permit an auxiliary function such as a derivative or antiderivative. In each case, you will be asked whether the current function(s) is (are) okay, then given a message such as

```
ENTER THE FUNCTION TO BE PLOTTED.  
Y(X) = .....
```

You can plot any function which can be expressed as a single Applesoft BASIC arithmetic statement. Writing an arithmetic statement in BASIC, as in any computer programming language, has similarities to and differences from writing in mathematical notation. The differences are most obvious in division, raising to a power, and grouping. For example,

suppose we wish to set

$$y(x) = \frac{x^2 - 2x + 1}{3x^3 + 4}$$

In BASIC, we would write

$$Y(X)=(X^2-2*X+1)/(3*X^3+4).$$

All operators and operands must be explicitly stated on one continuous line. The asterisk (*) must be used for multiplication and the carat (^) to show an exponent.

BASIC has a limited set of built-in functions. Those available in Applesoft BASIC are:

- SIN(X) = sine of X, X in radians;
- COS(X) = cosine of X, X in radians;
- TAN(X) = tangent of X, X in radians;
- ATN(X) = arc tangent of X, result in radians (principal value);
- INT(X) = the largest integer less than or equal to X;
- SGN(X) = the algebraic sign of X
(-1 for X < 0,
0 for X = 0,
+1 for X > 0).
- ABS(X) = the absolute value of X;
- SQR(X) = the positive square root of X,
X must be non-negative;
- EXP(X) = exponential function of X;
- LOG(X) = natural logarithm of X,
X must be positive.

The argument of the function must always be enclosed in parentheses.

You may also generate random numbers using RND(X). If X>0, the next number in a sequence of pseudo-random numbers is generated. If X=0, the most recently generated number is

repeated. If $X < 0$, a specified number is generated (the same number each time the same X appears).

Not all arithmetic operations in Applesoft BASIC are the same as in mathematics. Fractional powers are not defined for negative numbers. Therefore, if you wish to evaluate the cube root of X for negative X , you cannot use $X^{.33333}$, but you can use $\text{SGN}(X) * (\text{ABS}(X))^{.33333}$.

The precedence of operators in Applesoft BASIC is different from mathematics. Plotting $Y(X) = -X^2$ will plot $(-X)^2$, not $-(X^2)$.

The precedence of operators is as follows:

()	
+ - NOT	Unary operators
^	Exponentiation
* /	Multiply & divide
+ -	Binary operators (add and subtract)
> < >= <= => =< <> >< =	Logical relations
AND	Logical operators
OR	

In the chart above, operators are listed from highest to lowest priority; operators of the same priority are listed on the same line. In an expression, operators of the same priority are executed from left to right. (Applesoft II Reference Manual, page 36). If in doubt, use parentheses.

Applesoft BASIC permits us to combine into one statement certain function definitions which would take several lines in mathematics. Suppose, for example, that you wished to plot the

step function

$$Y(X) = \begin{matrix} 1 & X < 0 \\ 2 & 0 \leq X \leq 2. \\ 3 & X > 2 \end{matrix}$$

In Applesoft BASIC, you would write

$$Y(X)=(X<0)+2*(X>=0)*(X<=2)+3*(X>2).$$

The expressions in parentheses are logical expressions. If the expression is true, its value is one; if false, its value is zero.

In defining a function for ARBPLOT, you may use the independent variable name found in parentheses on the left side of the equal sign, the operators and built-in functions defined above, and explicit decimal constants. You may not use general representations for constants, such as letters of the alphabet. Your function definition may be approximately 50 characters long; it can spill over onto the next line. ARBPLOT checks to see that you have a legal function; it cannot tell whether you have the correct function for your application.

Many examples of functions are given in Chapters 3 and 5 and in the built-in examples in the programs.

Now it is time for you to try changing the definition of a function. In the main menu of Disk 1, select #1 as before. In the ARBPLOT1 menu, press "1" to change the function. Try $Y(X)=\text{SQR}(1-X^2)$. Be sure to press "RETURN" after you have entered the function. If you make a mistake in typing, you can use the left arrow key (←) to backspace and correct before pressing "RETURN". Leave all other settings the same, and press "7". (If you already were in ARBPLOT1, the limits should

be -1 and 2 with automatic scaling.) While calculating, an error message will appear, since $\text{SQR}(1-X^2)$ is not defined for $X>1$. Answer "N" to the question; instead of changing the function, press "2" to change the scaling. Accept the X-limits, but specify manual scaling with $0<Y<3$. Again, press "7" to plot the curve. At the end you will see $Y_{\min}=-1E+37$ and $Y_{\max}=1E+37$. These values indicate that the function was not defined for some X in your interval.

2.5. Saving the Display

You can save your display to diskette for use in a slide show, you can make a hard copy using a graphics-equipped printer, and you can modify the text on the display before saving. Chapter 4 discusses reasons for wanting to save the display and tactics for using slide shows and hard copy.

To try the procedures of this section, you will need one or preferably two blank diskettes for building your slide shows.

Having created the image you wish to save, exit from your plot producing program to the main menu by pressing "8". Your display is still in memory. Now press "9" to call the DISPLAY program. The following menu appears:

<<DISPLAY SAVER>>

DO YOU WISH TO

1. SAVE TO DISK FOR REGULAR SLIDE SHOW.
2. SAVE TO DISK FOR COMPRESSED SLIDE SHOW.
3. OUTPUT TO PRINTER.
4. LOAD IMAGE FROM DISK.
5. EDIT DISPLAY.
6. RETURN TO MAIN MENU.

WHICH (1-6)?

Your selection starts the display saving process.

2.5.1. Making a Slide Show

A slide show is a diskette containing display images with a program for showing them on the video screen. There are two types of slide shows, regular and compressed. For each, you must have a properly INITed (initialized) diskette. If you do not, answer "N" when asked, and the following instructions will appear on the screen:

TO PREPARE YOUR SLIDESHOW DISKETTE,
YOU MUST DO THE FOLLOWING:

1. REMOVE THE ARBPLOT DISKETTE.
2. INSERT BLANK DISKETTE & CLOSE DOOR.
3. TYPE 'INIT SHOW'. -or- TYPE 'INIT GRAPH PACK'.
4. WAIT 30 SECONDS UNTIL LIGHT ON DISK DRIVE GOES OUT & CURSOR APPEARS.
5. REMOVE SHOW DISKETTE. -or- REMOVE GRAPH PACK DISKETTE.
6. INSERT ARBPLOT DISKETTE & CLOSE DOOR.
7. TYPE 'RUN DISPLAY' TO GET BACK TO SAVING YOUR DISPLAY.

START WHEN THE CURSOR REAPPEARS

Insert a blank diskette as requested, and INIT your slide show diskette. Notice that the diskette for a regular slide show requires the SHOW program, while a compressed slide show requires the GRAPH PACK program. You cannot mix the two types of shows on one diskette.

Return to the DISPLAY program by typing RUN DISPLAY, and you are ready to continue building your slide show. Your display image has not been destroyed.

2.5.2. Regular Slide Show

A regular slide show consists of direct hi-res graphics screen images saved on diskette. Up to 13 images may be saved on one diskette. Each image must have a unique name.

At the DISPLAY menu, select #1. Insert an INITED SHOW diskette and confirm that you have the correct image. (Note: If you do not have an INITED SHOW diskette, create one from a blank diskette as described above.) When you give a name to your display, it is automatically saved to diskette and added to your slide show. If there is no more room for your image on the diskette or you have given the image a name you have already used, you will receive a warning message.

After you have saved your image (or had it rejected due to a full diskette) the image remains in memory. You may save it on another slide show diskette, compress it for a compressed slide show, print it, or use it as background for the next plot you create. These processes are described in the following

sections.

You can also put hi-res graphics images produced by other programs into your slide show. If the image is on another diskette, use Apple's FID program (on the DOS 3.3 master diskette) to transfer the image between diskettes. When the image is in memory, type "BSAVE SLIDENAME,A\$2000,L\$2000" if the image is on hi-res page 1, or "BSAVE SLIDENAME,A\$4000,L\$2000" if it is on hi-res page 2, where SLIDENAME is the name you wish to give the image.

If possible, store your images on the SHOW diskette in the order in which you expect to show them. You may show the slides in any order, but you will experience a ten-second delay any time you select slides out of order.

To show your regular slide show once you have created it, insert your SHOW diskette and boot the system, or type "RUN SHOW" if the system has been booted and is at Applesoft ("]" showing before the cursor). The following message appears:

<<ARB PLOT SLIDE SHOW>>

SHOW CREATES & RUNS A SLIDE SHOW OFF
ANY DISKETTE CONTAINING HI-RES IMAGES.

IF THE CATALOG IS LONG ENOUGH THAT
PREPARING SHOW
DOES NOT APPEAR, PRESS RETURN TO
FINISH CATALOG AND START PROCESSING.

IF CATALOG IS LONG WITH HI-RES IMAGES
NEAR BEGINNING, YOU MAY LOSE PICTURES.

You can use the SHOW program to run a slide show from hi-res images on any diskette, not just a SHOW diskette created by DISPLAY.

Now press "RETURN". The catalog appears and gradually disappears. SHOW is seeking the hi-res image files on disk and preparing the show from them. When the show is prepared, the following message appears (the slide names are from a particular show of curves in polar coordinates):

```
<<ARBPLOT SLIDE SHOW>>
YOUR SLIDES ARE:
A. TITLE PAGE
B. AP1
C. ALT CHAR SET
D. AP2-INTERP PROB
E. AL-LIMIT
F. AL-JUMP
G. SQUARE WAVE

PRESS SPACE BAR TO START WITH 1ST SLIDE
OR TO CONTINUE IN SEQUENCE.
TO CHOOSE A SPECIFIC SLIDE, PRESS ITS
LETTER.
PRESS 'RETURN' TO EXIT.
ANY OTHER KEY RETURNS TO THIS MENU.
YOUR CHOICE?
```

With your show prepared, press the space bar to display the first slide. Notice that the red light comes on, and the disk starts spinning, as soon as a new slide appears on the screen. The next slide in sequence is being loaded while you are viewing the current one. If you want a slide out of sequence, you must wait the ten seconds it takes to load the next slide before you can see it.

2.5.3. Compressed Slide Show

The Apple II stores a hi-res graphics image as 8192 bytes of bit-patterns describing the dots on the screen. If there are large blank areas on the screen, a compressing process can

save considerable storage space, both on disk and in memory.

The Harris GRAPH PACK uses a compressing process to save more images on a diskette, then builds a slide show of multiple images in memory at once for fast cycling while showing.

At the DISPLAY menu, select #2 and insert an INITED GRAPH PACK diskette in your disk drive. (Note: If you do not have an INITED GRAPH PACK diskette, create one from a blank diskette as described above.) The following menu will then appear:

SLIDE MENU:

1. COMPRESS AND SAVE A GRAPH
2. RECALL A COMPRESSED GRAPH
3. RECALL A STANDARD GRAPH
4. CREATE AND SAVE A SLIDE SHOW
5. RECALL AND DISPLAY A SLIDE SHOW
6. QUIT

YOUR CHOICE?

The entire GRAPH PACK process is in the GRAPH PACK program you have placed on your new diskette. Therefore, you can use it independently of the ARBPLOT diskettes for making a compressed slide show from hi-res images from any source.

Building a compressed slide show is a two-step process. Each time you have created a display you wish to include in a slide show, you COMPRESS AND SAVE it. When you have created and saved the images for your show, you next CREATE AND SAVE A SLIDE SHOW.

To save the graph in memory, at the SLIDE MENU select #1, COMPRESS AND SAVE A GRAPH. You will be asked whether your graph is in memory. If it is a display just created by an ARBPLOT program, it is in memory. You are then asked the page your graph is on. If it is an ARBPLOT display, it is on page 2. If in doubt where your image is, try either 1 or 2. You will be given an opportunity to verify that you really do have the image you want to save, before you save it.

(Note: If you wish to move a previously compressed graph to memory, or compress a regular hi-res image residing on disk, at the SLIDE MENU select #2 or #3. You must place the diskette containing the image in the disk drive.)

To save the compressed graph, you are told to "INSERT DISK ON WHICH GRAPH IS TO BE SAVED AND PRESS RETURN". Generally, you do nothing here; your GRAPH PACK diskette is the one which will store your compressed graphs. However, you can if you wish build a library of compressed graphs on one diskette with completed slide shows on another.

With your storage disk in place, you are shown the names of graphs already stored (none if this is the first graph you have created) and asked for the name of your graph. Then the graph is stored, after which you may return to the DISPLAY menu (ARBPLOT), to the main GRAPH PACK SLIDE MENU, or exit to BASIC.

The compression process depends upon a display image having lots of blank space in it. It saves only the non-blank bytes of an image, but must save their locations. Therefore, if less than half the bytes are blank, the "compressed" version will be larger than the original. The busier the image, the less likely it is that compressing will save space.

After you have compressed the graphs you want, you are ready to CREATE AND SAVE A SLIDE SHOW. At the SLIDE MENU, select #4. (To reach the SLIDE MENU to create a show if you are not already in GRAPH PACK, select "9" at the main ARBPLOT menu, then "2" at the DISPLAY menu, or boot your GRAPH PACK diskette directly.) The following menu appears:

CREATE AND SAVE A SLIDE SHOW
SECTORS REMAINING: 56

CHOOSE AN OPTION:

1. LOAD A GRAPH.
2. SHOW GRAPHS LOADED SO FAR.
3. SAVE SLIDE SHOW.
4. CLEAR SLIDE SHOW IN MEMORY.
5. RETURN TO MENU.

YOUR CHOICE?

The first step in creating a slide show is to load the graphs. Select #1, LOAD A GRAPH. On the screen you will see "# SECTORS / NAME:" where NAME is the name you gave to each compressed graph as you saved it, and # SECTORS is the length of the compressed graph in disk sectors. Now note the SECTORS REMAINING which are initially 56. Your total compressed slide show can be no longer than 56 sectors. Each time you load a graph the number of sectors remaining is reduced, but only after loading. It is up to you to check that there is room for the next graph. If you try to save too much you may wipe out everything in memory and have to reboot the system and start over. When SECTORS REMAINING is less than 10, the number will flash as a warning.

You can pick up a compressed graph from any diskette on which it resides. To switch to a new source of graphs, you must insert the new diskette before pressing "1" in the CREATE AND SAVE menu.

As you are creating your compressed slide show, you may wish to check your progress. Press "2" to see the graphs already loaded. The "ESC" key returns you to the CREATE AND SAVE menu.

When you have created the slide show, you must save it. Press "3" to save. You will be asked to insert the diskette on which you wish to save the show. The diskette may be (but need not be) the one on which you have your compressed graphs. You will be shown the names of slide shows already on the disk, then you will be asked to name your new show. If you make a mistake (disk full, disk write locked, etc.) the question will appear again. Correct the indicated problem and continue. When the show is saved, you are returned to the CREATE AND SAVE menu.

At this point, you may have compressed so many graphs that you need (or wish) to make more than one slide show. If so, press "4" in the CREATE AND SAVE menu to clear the memory. SECTORS REMAINING returns to 56 and you are ready to start creating the next show.

Rather than saving a new show, however, let's run the show you have already created. Press "5" to return to the SLIDE MENU, and select #5, RECALL AND DISPLAY A SLIDE SHOW. You are shown the names and sizes of the slide shows on the disk and asked to enter your choice. If your answer is wrong, e.g., you misspelled the name, the question will appear again. Once your

show is loaded, press the space bar repeatedly to cycle the show. Notice how fast you can cycle, which is useful if you are showing successive stages of a process, like the effects of increasing number of subdivisions in numerical integration. Now press "ESC" to return to the menu. You are back at the SLIDE MENU where you can run another show.

2.5.4. Output to Printer

You may wish to save a display on paper for inclusion in a report or class notes, or to make a transparency for an overhead projector.

Many printers for the Apple II computer have graphics capability. ARBPLOT has provision for saving your display on paper on three of the more popular printers. At the DISPLAY menu, select #3, OUTPUT TO PRINTER. The following menu appears:

<<HARDCOPY DISPLAY>>

YOU CAN MAKE A PERMANENT COPY OF YOUR
DISPLAY ON CERTAIN PRINTERS.

DO YOU HAVE

1. APPLE SILENTYPE
2. PAPER TIGER W/GRAPHICS OPTION
3. MX-80 W/GRAPPLER
4. OTHER

WHICH (1-4)?

The Apple Silentype driver works automatically for the controller in any slot. The Paper Tiger and MX-80 drivers ask for the slot number of your controller. You have your choice of two picture sizes for the MX-80.

Be sure your printer is ON and has paper in it!! Then enter the appropriate number listed for your printer.

When printing is complete, you automatically return to the DISPLAY menu. The display image is still in memory, available for other processing.

Output to the Paper Tiger or MX-80 gives a large well-proportioned picture suitable for making a transparency for an overhead projector. The output to the Silentype printer, on the other hand, is compressed to about 80% of the vertical dimension of the picture seen on the screen. In most cases, the squashed display is acceptable or can be adjusted by using manual scaling to vary the proportion of the plot. However, to get true proportions in polar coordinates, you must change the plot program. In ARBPLOT3 and ARBINT6, insert line 24047 FF=1.

Suppose you have a different printer with graphics capability, a different controller, or wish to use a different hi-res screen-dump program. Then what do you do?

You have three choices. First, you can save your image on disk for a regular slide show, then after you have finished a session with ARBPLOT you can use your software to recall the image from disk and print it. This approach is necessary if your software requires an image on hi-res page 1.

Second, if your printer driver program can work with an image on hi-res page 2, display your image and then press "0"

to exit at the main ARBPLOT menu, rather than "9", SAVE DISPLAY. Remove the ARBPLOT diskette, insert the diskette with your program on it, and run your program. When you have finished, remove your diskette, insert the ARBPLOT diskette and type "RUN APMENU."

Third, it may be possible to incorporate your printer program into ARBPLOT. See Chapter 6 for suggestions on doing this.

2.5.5. Loading an Image from Disk

There are several reasons you might wish to load a display image from disk. You might be rearranging slide shows, including a non-ARBPLOT display in a slide show, recalling an image to use as background for a new ARBPLOT display (see Fourier Series example in Chapter 5), printing, etc. To load an image from disk, select #4 at the DISPLAY menu.

You will be asked for the file name under which the image was saved on disk and the slot and drive in which you have inserted the diskette. If you have a single drive system, the slot will be "6" and the drive "1"; you must remove the ARBPLOT diskette and insert the diskette with your display image before loading the image, then remove your diskette and insert the ARBPLOT diskette before taking the next action at the DISPLAY menu. The image in memory for processing is now the one you have loaded from a diskette.

Loading a compressed image is accomplished through RECALL COMPRESSED GRAPH within GRAPH PACK.

2.5.6. Editing the Display

MH GRAPH EDITOR lets you add, delete, or change text on any ARBPLOT display or any other hi-res image. It offers upper and lower case, special symbols, and an "exclusive-or" mode for printing text in the color (black or white) opposite that of the background. The image to be modified must be in memory (it is, if you are editing an ARBPLOT display you have just created). The program saves a back-up of the original display on the unused graphics page which can be restored at any time, thus allowing you to experiment without losing the original.

Once your image is in memory, whether created by an ARBPLOT program or loaded from disk by DISPLAY or GRAPH PACK, enter MH GRAPH EDITOR by selecting #5 in the DISPLAY menu. You can also use MH GRAPH EDITOR outside ARBPLOT by loading your graph, then inserting the ARBPLOT diskette, and typing "RUN MH GRAPH EDITOR".

The program starts with three pages of instructions in the editing functions. These functions are described below and can also be recalled to the screen at any time while you are editing. You will be shown the two hi-res pages to select the one you wish to edit, then the editing session will begin. You will see the graph you have selected, and a flashing cursor will indicate the current position.

There are four types of editing actions: cursor movement, upper/lower case change, normal or exclusive-or text addition, and normal or alternative character set. At any time Ctrl-E allows you to exit from the editing session.

Cursor Movement. The cursor can be moved, without affecting the graph, in one of five ways:

"→" Move the cursor right one space.
"←" Move the cursor left one space.
Ctrl-Q Move the cursor up one space.
Ctrl-Z Move the cursor down one space.
RETURN Move cursor to start of next line.

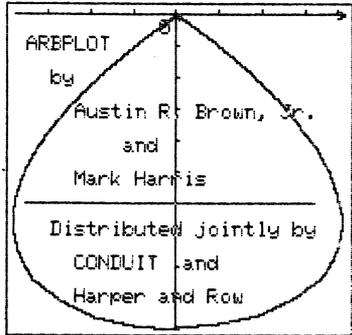
Upper Case and Lower Case. The "ESC" key will cycle through three modes. When you first enter the editor, pressing a letter key will result in lower case printed at the cursor position. If you depress "ESC" first, however, the next letter key you press will show in upper case, and subsequent letters will revert to lower case. Pressing "ESC" twice (from the lower case mode) results in a shift lock and all further letters show in upper case until "ESC" is pressed again.

Exclusive-Or Mode. Normally, if the cursor is flashing over a portion of the graph and you type a character, a cursor-sized portion of the graph is lost. If you then erase the character, the graph from that position does not re-appear. (The entire original graph can be restored as described below, but the alteration from the editing session will have been lost.) The exclusive-or mode, entered by pressing Ctrl-X, will show each letter as the reverse of its background, so that typing the same letter twice in the same position will restore that portion of the graph to its original appearance. This allows experimentation without destruction. The exclusive-or mode can also be used to create text which doesn't disconnect a graph or otherwise get in the way. Once entered, this mode stays in effect until you press Ctrl-N (normal mode), or until you end the editing session. Exclusive-or mode was used in the figure below; notice the mingling of curve and characters.

```

ARB PLOT2
Tmin=-1
Tmax=1
Xmin=-.384
Xmax=.384
Ymin=-1
Ymax=0
ΔX=.1
ΔY=.1
T=1
X = T^3-1
Y = T^2-1

```



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Alternate Character Set. In addition to using the standard characters represented on the Apple II keyboard, you can access an alternate character set by preceding any letter "a" through "t" with Ctrl-A. (The Ctrl-A prefix must precede each letter for which an alternate is desired.) The alternate character set and corresponding letters are given below:

Alternate character set (Ctrl-A)

A	·	K	∫
B	Δ	L	∫
C	Σ	M	∫
D	/	N	∫
E	←	O	∫
F	→	P	∫
G	↓	Q	∫
H	↑	R	∫
I	√	S	∫
J	π	T	∫

Ending the Editing Session. To quit editing the graph, temporarily or for good, type Ctrl-E. You will exit the graphics page to the following menu:

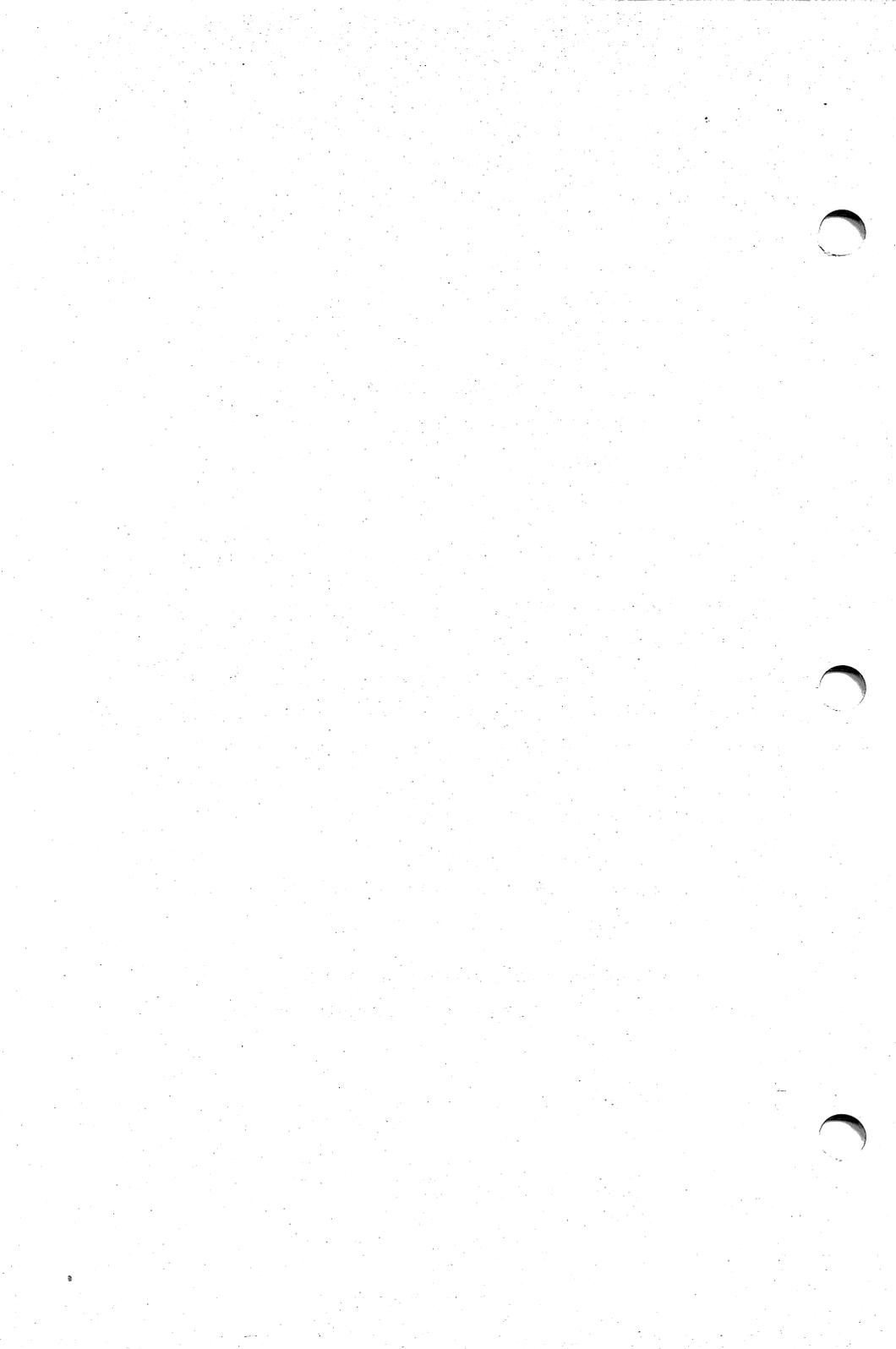
CHOOSE AN OPTION:

1. SAVE GRAPH
2. EDIT PICTURE
3. RE-EDIT ORIGINAL PICTURE
4. REVIEW INSTRUCTIONS
5. EXIT

YOUR CHOICE? (1-5)

SAVE GRAPH returns you to the DISPLAY program menu from which the edited graph can be saved to disk, compressed, or printed. EDIT PICTURE picks up where you left off editing a graph; nothing will have changed. RE-EDIT ORIGINAL PICTURE restores your original graph from the alternate hi-res page (wiping out the edited version) and puts you into the edit mode for a fresh start. REVIEW INSTRUCTIONS returns you to a description of program functions, from which you can continue editing the graph. EXIT returns you directly to the main ARB-PLOT menu or Applesoft.

All the graphics text generation in ARBPLOT uses the High Resolution Character Generator by Christopher Espinosa.



3. Details: What Each Program Does and How to Use It

3.1. Curve Plotting Programs

The first four programs on Disk 1 plot curves. As shown on the main ARBPLOT menu, these are

1. PLOT Y(X) (Program ARBPLOT1)
2. PLOT X(T) VS. Y(T) (Program ARBPLOT2)
3. PLOT R(T) (POLAR) (Program ARBPLOT3)
4. PLOT Y1(X), Y2(X), SIMULTANEOUSLY (Program ARBPLOT4)

ARBPLOT1 plots Y as a function of X in rectangular coordinates. ARBPLOT2 plots X(T) vs. Y(T) (parametric equations) in rectangular coordinates. ARBPLOT3 plots R (radius) as a function of T (angle theta) in polar coordinates. ARBPLOT4 plots two independent curves Y1(X) and Y2(X) in rectangular coordinates. All share certain attributes as described in the example from ARBPLOT1 given above; each has its own special features.

Each of the plot programs can have automatic or manual scaling. In automatic scaling, all values of the function are calculated before plotting; the plot fills the screen. In manual scaling you set the minimum and maximum values for the dependent variable(s). You must use manual scaling if the function is unbounded or has a gap in its domain.

Each plot program requires you to specify the increment between points in the independent variable as well as the interval for the plot. When entering limits, you can press "RETURN" to retain the previous value. The increment in ARBPLOT1, ARBPLOT2, and ARBPLOT4 is the actual difference between successive values of the independent variable. ARBPLOT3 calls for the number of points per revolution. The resolution of the horizontal axis in rectangular coordinates in ARBPLOT is 190 points; therefore, there is no reason to plot more than 190 points in ARBPLOT1 or ARBPLOT4 (the program limit is 200 points). However, up to 500 points may be plotted in ARBPLOT2 (parametric) or 768 points in ARBPLOT3 (polar) to give adequate resolution for curves which double back upon themselves. The limits are the dimensions of the arrays of function values filled when automatic scaling is used.

All of the plot programs permit you to clear or keep the previous display. Keeping the display lets you overlay curves on one another. If you keep the previous display, all of the text on the screen refers to the last curve plotted. In all of the plot programs except polar coordinates, you may plot the curve as a dotted or solid line. In all of them, you may show or suppress the axes. Pressing the appropriate key switches you back and forth between the two alternatives.

If you use a solid line to plot a curve with finite jumps, such as $Y(X)=INT(X)$, a line will be drawn across the gap rather than skipping it. Since ARBPLOT only calculates a finite number of function values, it cannot distinguish between a steep rise and a discontinuity. To show a finite discontinuity, plot a dotted line with minimum spacing between points.

You may interrupt the plotting at any time by pressing any key. Press a key a second time to resume plotting. Pressing Ctrl-E as the second stroke does not resume plotting, but instead moves directly to printing minimum and maximum values, then takes you to "End" to return to the menu.

3.1.1. Special Features of ARBPLOT1

ARBLOT1 plots a single function of a single variable in rectangular coordinates. In a sense, ARBPLOT1 has no special features; it is the basic plot program, and its features were described in the examples in Chapter 2. However, let's try one more example to illustrate features of all of the programs.

At the main ARBPLOT menu, select #1, PLOT Y(X). Set $Y(X)=INT(X+1)$, $Xmin=0$, $Xmax=3$, and $DELTA X=.1$, with automatic scaling. Clear the previous display, choose a solid line for the curve, and show the axes. Now press "7" to plot.

First, note the rising lines connecting the horizontal segments as mentioned above. If you want to show the discontinuities, set $DELTA X=.015$ and use a dotted line for the curve. The points touching each other give the effect of a solid line.

Next, notice that the bottom of the display area is $Y=1$, so the X-axis is not shown and the tic-marks appear on the border. If you want the display to start at $Y=0$, use manual scaling with $Ymin=0$.

Finally, notice that $Ymax=3$, even though $Y(3)=INT(3+1)=4$. Since arithmetic in Applesoft BASIC is binary, carrying a

finite number of digits, roundoff errors in computing the increment and calculating successive values of the independent variable may lead to overshoot or undershoot at the end of the interval.

3.1.2. Special Features of ARBPLOT2

ARBPLOT2 plots two parametric equations against each other. The main additional feature it has over ARBPLOT1 is the ability to automatically make horizontal and vertical scaling the same or independent of each other.

>>ARBPLOT2 - CURRENT VALUES & CHOICES<<

1. FUNCTIONS

$X(T) = T^3 - T$
 $Y(T) = 3 * T^2 - 1$

2. LIMITS/SCALING DELTA T= .05

 TMIN=-1 TMAX=1
 XMIN=0 XMAX=0
 YMIN=0 YMAX=0
 AUTOMATIC :SCALING: INDEPENDENT

3. SCREEN: CLEAR PRESENT DISPLAY

4. CURVE: SOLID LINE

5. AXES: SHOW

ACTIONS:

6. SHOW CURRENT DISPLAY

7. PLOT CURVE

8. EXIT

CHOICE: CHANGE VALUE(1-5)

 ACT(6-8)

 WHICH(1-8)?

Run the built-in example in ARBPLOT2 to illustrate its use. At the main ARBPLOT menu, select #2, PLOT X(T) VS. Y(T). In the ARBPLOT2 menu, press "7". The figure drawn looks like an aspen leaf, filling the screen from left to right and top to

$x_{max}=.5$, and independent scaling on the axes, then press "7" to

Then set scaling to manual, $x_{min}=-.5$, $x_{max}=1.5$, $y_{min}=-1$,

First, run with automatic scaling to see the entire figure.

$X(T)=SIN(7*T)$, $Y(T)=SIN(8*T)$, $T_{min}=0$, $T_{max}=6.3$, DELTA T=.025.

Manual scaling lets us examine a portion of a curve. Set

accurately.

while same scaling shows the proportions of the ellipse more

independent scaling draws an ellipse that is nearly circular

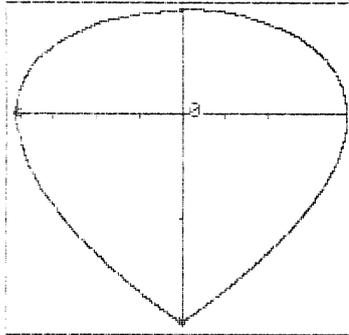
$T_{min}=0$, $T_{max}=6.3$, DELTA T=.1 and leave scaling automatic.

Try another example. Set $X(T)=3*COS(T)$, $Y(T)=2*SIN(T)$,

```

AREFLDZ
Tmin=-1
Tmax=1
Xmin=-.884
Xmax=.884
Ymin=-1
Ymax=2
DeltaX=.1
DeltaY=1
T=1
X = 1.93-1
Y = 3*1.72-1

```



axes.

of the screen. Minimum and maximum values are the same on both

now is narrower and looks like a willow leaf on the left side

Again press "7" to calculate and sketch the curve. The curve

change to same scaling, leaving all other settings the same.

separately and independently scaled to fill the screen. Now

automatic and independent. Each dependent variable was

bottom. Return to the menu and note that the scaling was

plot. Note how the odometer keeps running (and the program keeps calculating) even when offscale. The minimum and maximum values printed to the left of the curve are the extreme calculated values, not displayed values.

Next, change to same scaling on both axes, leaving all other settings alone; the entire curve now appears. Note that with manual scaling there is no initial waiting period for calculation; at times you may find it useful to develop a curve outside of class using automatic scaling, note the limit settings, and run it with manual scaling in class. As a final exercise, set DELTA T=.1 with no other changes, then plot and note the tradeoff between speed and smoothness of the curve.

3.1.3. Special Features of ARBPLOT3

ARBLOT3 plots a single function, R , of a single variable, T , in polar coordinates. The independent variable is given in radians. ARBPLOT3 differs from the other plot programs in that the increment is given in points per revolution rather than DELTA T, and that the curve is always a solid line when R is positive and dotted when R is negative.

At the main ARBPLOT menu, select #3, PLOT R(T) (POLAR). When the ARBPLOT3 menu appears, observe that there are only four values to be set.

>>ARBPLT3 - CURRENT VALUES & CHOICES<<

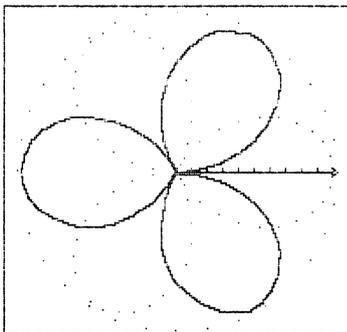
1. FUNCTION
 $R(T)=\text{SIN}(1.5*T)$
 2. LIMITS/SCALING POINTS PER REV= 64
 TMIN=0 TMAX=12.6
 RMIN=0 RMAX=0
 SCALING: AUTOMATIC
 3. SCREEN: CLEAR PRESENT DISPLAY
 4. AXES: SHOW
- ACTIONS:
6. SHOW CURRENT DISPLAY
 7. PLOT CURVE
 8. EXIT

CHOICE: CHANGE VALUE(1-4)
 ACT(6-8) WHICH(1-8)?

Press "7" to run the example. Note that only the positive polar axis is displayed. As the curve is drawn, a "+" periodically flashes in a circle around the rim of the plot showing the progression of the angle Theta (T). As T increases, R goes from zero to one to zero, while a solid line is drawn. As R becomes negative the curve changes to dotted and the flashing "+" is across the circle. When R once again becomes positive the curve becomes a solid line.

ARBPLT3
Tmin=0
Tmax=12.6
Rmin=-1
Rmax=1

$\Delta R=.1$
T=12.6
 $R = \text{SIN}(1.5*T)$



Now let's try plotting a lemniscate, a function with a gap in its domain. Set $R(T)=\text{SQR}(\text{COS}(2*T))$, $T_{\text{min}}=0$, $T_{\text{max}}=6.3$, points per revolution=80, but leave scaling automatic. Press "7" to plot. An error message will appear while calculating since the cosine goes negative. Answer "N" to return to the ARBPLOT3 menu and change to manual scaling with $R_{\text{min}}=0$ and $R_{\text{max}}=1$. Again plot. Observe that the second cusp of the lemniscate does not start at zero. This is due to roundoff error from the finite number of points plotted. Experiment with changes in the number of points per revolution and tiny changes in T_{min} to see how close you can come to showing the whole curve. Note also that $R_{\text{min}}=-1\text{E}+37$ and $R_{\text{max}}=1\text{E}+37$, the signal that there have been points at which R could not be evaluated.

Return to the ARBPLOT3 menu for one more example, this time a hyperbola to show a function increasing without bound. Set $R(T)=1/(1+1.5*\text{COS}(T))$, $T_{\text{min}}=0$, $T_{\text{max}}=6.3$, points per revolution=160, manual scaling, $R_{\text{min}}=-4$ and $R_{\text{max}}=4$. When the plot is completed note that R_{min} and R_{max} are not infinite. The finite number of points with finite arithmetic usually will skip discontinuities; R_{min} and R_{max} will be $1\text{E}+37$ if the discontinuity should be reached exactly.

To plot entirely solid or entirely dotted line curves in polar coordinates turn to parametric equations. Use ARBPLOT2 to plot $X(T)=R(T)*\text{COS}(T)$ vs. $Y(T)=R(T)*\text{SIN}(T)$ where $R(T)$ is your polar function.

3.1.4. Special Features of ARBPLOT4

ARBPLOT4 plots two independent functions, $Y1(X)$ and $Y2(X)$, of a single independent variable in rectangular coordinates.

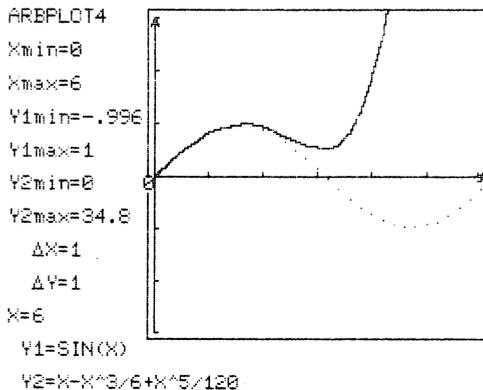
This is useful in the study of the behavior of two related functions. Additional features include plotting a function with its inverse and automatically drawing vertical lines through the zeroes of $Y_2(X)$. At the main ARBPLOT menu, select #4, PLOT $Y_1(X), Y_2(X)$, SIMULTANEOUSLY.

>>ARBLOT4 - CURRENT VALUES & CHOICES<<

1. FUNCTIONS
 $Y_1(X) = \sin(X)$
 $Y_2(X) = X - X^3/6 + X^5/120$
 2. LIMITS/SCALING DELTA X = .2
 XMIN=0 XMAX=6
 YMIN=-3 YMAX=3
 SCALING: MANUAL
 3. SCREEN: CLEAR PRESENT DISPLAY
 4. CURVES: Y1 IS DOTTED LINE
 Y2 IS SOLID LINE
 NO VERTICALS ON Y2
 5. AXES: SHOW
- ACTIONS:
6. SHOW CURRENT DISPLAY
 7. PLOT CURVES
 8. EXIT
- CHOICE: CHANGE VALUE (1-5)
 ACT (6-8)

WHICH (1-8)?

Press "7" to run the example, the fifth degree Taylor approximation to $\sin(X)$.



Scaling is manual. Note that one curve goes off scale while the other doesn't. Observe that Ymin and Ymax on the display are the calculated range of Y while Ymin and Ymax in the menu are the screen bounds. With the same functions, set Xmin=0, Xmax=4, DELTA X=.1, and automatic scaling. Now plot and observe that the two functions have the same scaling, from the minimum of Y1 to the maximum of Y2.

Gaps in the domains of the two functions are treated independently. Set $Y1(X)=\text{SQR}(1-X^2)$, $Y2(X)=\text{SQR}(X^2-1)$. Set Xmin=-2, Xmax=2, DELTA X=.05, manual scaling, Ymin=0, Ymax=1.5. Plot; note that for any X except X=-1 and +1, only one of the curves exists.

You can create and plot the inverse of a function. Set $Y1(X)=\text{EXP}(X)$, $Y2(X)=\text{INVERSE}$, Xmin=-1, Xmax=4, DELTA X=.1, manual scaling, Ymin=-1, Ymax=4, then plot. Manual scaling is needed since the process interchanges X and Y in plotting the inverse. The function need not be monotonic; try $Y1(X)=\text{SIN}(X)$ on the same interval to show the possible need for restricting the domain in order to have an inverse.

Finally, let $Y1(X)=\text{SIN}(X)$, $Y2(X)=\text{COS}(X)$, Xmin=0, Xmax=6, DELTA X=.1, automatic scaling, and auto verticals (see choice 4). As the curves are plotted, vertical lines are drawn automatically through the zeroes of Y2. This feature can be used to show relationships between a function and its derivative. Compare this feature in ARBPLOT4 with the similar one for a geometrically generated derivative in MH PLOT1 (selection 6, Disk 1).

3.2. Programs Involving Derivatives

Three programs on Disk 1 involve limits, derivatives and their use. As shown on the main ARBPLOT menu, these are:

- 5. LIMITS & DERIVATIVES (Program ARBLIM)
- 6. DERIVATIVE FUNCTION WITH EXTREMA (Program MH PLOT1)
- 7. ROOTS OF $F(X)=0$ (Program ARBROOT)

ARBLIM presents the epsilon-delta definition of the limit, the slope of the tangent as the limit of slopes of secants, and the mean value theorem for derivatives. MH PLOT1 shows the geometric relationship between a function and its derivative. ARBROOT approximates roots of $F(X)=0$ by Newton's Method, Regula Falsi, and Bisection. As with the plotting programs, each program permits you to specify the function to be plotted and the interval of plotting, then carries out its appropriate process.

3.2.1. Limits and Derivatives

ARBLIM presents graphical demonstration of three limit problems: the epsilon-delta definition of the limit of a function, the slope of the tangent as the limit of the slopes of secants, and the mean value theorem for derivatives. In the main ARBPLOT menu select #5, LIMITS & DERIVATIVES.

>>ARBLIM - CURRENT VALUES & CHOICES<<

1. FUNCTION
 $Y(X)=X^3$
2. LIMITS
XMIN=0 XMAX=2
3. PROBLEM: DERIVATIVE AT X=1
4. SCREEN: CLEAR PRESENT DISPLAY
5. DERIVATIVE:
 $Y'(X)=3*X^2$

ACTIONS:

6. SHOW CURRENT DISPLAY
 7. PLOT CURVE
 8. EXIT
- CHOICE: CHANGE VALUE(1-5)
ACT(6-8)

WHICH(1-8)?

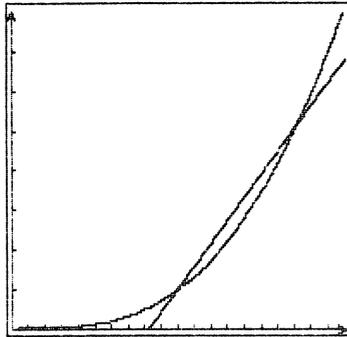
As with other programs, you must specify the function to be plotted and the limits on the independent variable. You do not specify DELTA X; 49 points are automatically plotted with automatic scaling. In addition, you must select the problem to be solved, which is reached through a subsidiary menu by pressing "3". Don't do this yet.

To see ARBLIM at work, press "7" at the ARBLIM menu. The problem is to find the derivative of $Y(X)=X^3$ at $X=1$. The curve will be plotted for $0 \leq X \leq 2$, then the secant through $X=1$ and $X=2$ will be drawn. When the message "L,R,<,>,E?" appears at the bottom of the screen press "<" to move the point of crossing of the secant to the left, repeating the process until you have reached the tangent, DELTA-X=0. If you want to start the limiting process again from the left, press "L" or from the right, press "R". To exit the menu, press "E".

```
ARBLIM
Derivative
Xmin=0
Xmax=2
Ymin=0
Ymax=8
```

```
ΔX=.1
ΔY=1
```

```
Y = Y^3
DELTA-X= .75      SLOPE= 5.81
Y'(1)=3           (EXACT)
```



You can overlay one plot on another by keeping the previous display when solving a new problem. You may, but need not, specify the derivative function for comparison with the limit in the derivative and mean value problems. Warning: ARBLIM does not check to see whether your specified $Y'(X)$ really is the derivative function.

3.2.1.1. Limit

The epsilon-delta definition of the limit of a function at a point states that L is the limit of $Y(X)$ as X approaches XL if given $\epsilon > 0$ there exists $\delta > 0$ such that $0 < |X - XL| < \delta$ implies $|Y(X) - L| < \epsilon$. ARBLIM checks only $|X - XL| < \delta$. (Note: In the following discussion, EPS stands for ϵ and DELTA stands for δ .)

Press "3" in the ARBLIM menu to select the problem to be solved. You will now have the following choices:

- - PROBLEM - -

PRESENT CHOICE IS DERIVATIVE

THE CHOICES ARE:

1. LIMIT
2. DERIVATIVE
3. MEAN VALUE

WHICH(1-3)?

Choose "1" for the epsilon-delta definition of the limit. You are asked for the limit. Ignore this; we are making more changes.

The function must be increasing or decreasing with no maxima or minima internal to the plotting interval. Set $Y(X)=X^3+1$ for illustration. Ignore the request for the limit. Note that changing the function wipes out the derivative function, which is not needed in this case.

Set $X_{min}=0$ and $X_{max}=1$. You will again be asked for the value of the limit. Reject the given value, then answer "Y" to get $Y(0.5)$, or else specify a value, e.g., specify $Y=0.2$ as your guess for the limit. You will have noted that you have been asked for the limit three times. You will be asked for the limit any time you change the problem, the function, or the interval. Specify the limit in the last change you make.

Now press "7" to plot the function and begin the limiting process. The curve $Y=X^3+1$ is plotted for $0 \leq X \leq 1$, scaled so that vertical and horizontal lines in the middle of the screen show the specified limit point, with the location of the point given numerically at the bottom of the screen. Horizontal lines are drawn at $Y=LIMIT+EPS$, then the location of the vertical lines at $X+DELTA$ are calculated and displayed so that

|X-XL|<DELTA forces |Y-LIMIT|<EPS. EPS and DELTA are then displayed numerically.

The initial value of EPS is 1/4 the height of the screen. Choices given at the end of each step are "C,Z,E?". Choosing "C" divides EPS by two. "Z" zooms to fill the screen with the previous EPS-DELTA rectangle. "E" exits to the ARBLIM menu.

The value of DELTA is obtained by interpolation in the table of 49 values of Y which define the curve. Therefore, EPS cannot be made arbitrarily small on a given display. When DELTA reaches 1/24 of the screen width the message "AS CLOSE AS POSSIBLE; Z TO ZOOM" is displayed. You can zoom to make EPS smaller, or you can exit. If LIMIT is not the limit, you eventually will get the message "*** NOT THE LIMIT ***" when there is no DELTA to go with EPS. (Try this example again with LIMIT=0.2.)

With very small EPS, on the order of 10^{-5} or smaller, you may get "*** NOT THE LIMIT ***" or even "Y(X) MUST INCREASE OR DECREASE/NO UPS AND DOWNS" even though you have a proper function and correct limit, due to roundoff error in the finite arithmetic of the computer.

A function with a finite jump can be presented, e.g., $Y(X)=X*\text{INT}(X+1)$ for $X_{\min}=0$, $X_{\max}=2$, even though the jump may show as a steep line rather than a discontinuity. Try $Y(1)$ for the limit, then $Y=1.5$. If the jump between successive points is greater than 1/4 the height of the screen, the jump is assumed to be due to a discontinuity, and a gap is shown. You will quickly get "*** NOT THE LIMIT ***" for any guess for the limit of a function with a jump. Due to a peculiarity in the program, you will sometimes get "Z TO ZOOM" once before

*** NOT THE LIMIT ***.

You can begin to show that the limit of the function is not necessarily the value of the function if the jump is small, although zooming will soon reveal the jump. Try $Y(X)=(X \leq .999 \text{ OR } X \geq 1.001) * X * X + (X > .999 \text{ AND } X < 1.001) * 1.2$ for $0 \leq X \leq 2$. Roundoff error requires us to say $(X > .999 \text{ AND } X < 1.001)$ rather than simply $X = 1$.

3.2.1.2. Derivative

The built-in example in ARBLIM finds the slope of the tangent as the limit of the slopes of secants. Let's try another example. Press "3" to select the problem to be solved, then press "2" for the derivative.

The function must be defined on the entire plotting interval and differentiable on some interval containing the midpoint. Peculiar things can happen with an improper function. Set $Y(X) = X^3 - X$ for illustration. Note that changing the function wipes out the previous derivative function. Set $X_{\min} = -1$ and $X_{\max} = 1.5$. The derivative will be approximated at $X = 0.25$.

Now press "7" to plot the function and begin the limiting process. The curve $Y = X^3 - X$ is plotted for $-1 \leq X \leq 1.5$ scaled to fill the screen from bottom to top, then the secant through $X = 0.25$ and $X = 1.5$ is drawn, and the message "L,R,<,>,E?" appears at the bottom of the screen. Choosing "L" draws the secant to the left edge of the curve, and "R" to the right edge. Choosing "<" moves the point of crossing one unit to the left, and ">" one unit to the right. The current value of ΔX and the slope of the secant are shown at each step. When $\Delta X = 0$

the program draws an approximation to the tangent by drawing the secant with base width $.001*(X_{max}-X_{min})$ from the left or right. Twelve steps span the distance from full left or right to the middle.

Choosing "E" exits to the ARBLIM menu. Now exit to the menu and set the derivative $Y'(X)=3*X^2-1$. Press "7" again and repeat the limiting process. The exact (to 3 significant digits) value for the derivative shows at the bottom of the screen. Warning: ARBLIM does not check to see whether your specified $Y'(X)$ really is the derivative function.

You can get a rough idea of the behavior of a function with a vertical tangent or with left and right limits of the secants unequal. Try $Y(X)=\text{SQR}(\text{ABS}(X))$ for $X_{min}=-1$ and $X_{max}=1$.

3.2.1.3. Mean Value Theorem

The Mean Value Theorem states that there is at least one point on the arc of the curve of a differentiable function at which the tangent is parallel to the chord. Let's demonstrate this. Press "3" to select the problem to be solved, then press "3" for Mean Value.

The function must be defined and differentiable on the entire plotting interval. Peculiar things can happen with an improper function. Set $Y(X)=4*X-X^2$ for illustration. Note again that changing the function wipes out the previous derivative function. Set $X_{min}=0$ and $X_{max}=3$.

Now press "7" to plot the function and begin the limiting process. The curve $Y=4*X-X^2$ is plotted for $0 \leq X \leq 3$ scaled to fill the screen bottom to top, the chord is drawn, the first

secant is drawn, and the message "L,R,<,>,E?" appears. The X-coordinate of the left-most intersection of chord and curve appears, as does the slope of the chord. Pressing ">" moves the point of intersection to the right, "L" to the extreme left, and "R" to the extreme right. As before, "E" exits to the ARBLIM menu.

A 25-point (24 intervals) grid is used. When stepping reaches the grid point closest to the point of tangency, a new process is invoked. A parabola is fitted to the nearest point and its neighbors, the point of tangency to that parabola is determined, and a vertical line is drawn through the point. The point of tangency to the parabola is taken as the approximation to the point of tangency to the curve.

Now return to the ARBLIM menu with "E" and enter the derivative $Y'X(X)=4-2*X$. Repeat the plotting/limiting process by pressing "7" followed by the necessary number of ">"s. When tangency is reached, the message "Y'(1.5)=1" appears at the bottom of the screen. The derivative has been evaluated at the approximate tangent point.

To see that the tangent point is only approximate and to see multiple tangents, repeat the Mean Value process with $Y(X)=X^3-X$ and $Y'(X)=3*X^2-1$ for $-1 \leq X \leq 1.5$.

3.2.2. Derivative Function with Extrema

MH PLOT1, selection #6 on ARBPLOT Disk 1, geometrically illustrates the relationship between a function and its derivative. The program is intended primarily for classroom demonstration of (1) the derivative as a function and (2) the

first derivative test for extrema. You need only enter the function and its domain (and its range if manual scaling is selected); the program computes the derivative numerically, and proceeds on its own.

The program first takes you from the idea of derivative at a point (tangent slope) to the idea of the derivative function. Then based on changes of sign of the derivative, local extrema in the domain are computed.

To see how MH PLOT1 works, on the main ARBPLOT menu select #6, DERIVATIVE FUNCTION WITH EXTREMA.

>>MH PLOT1 - CURRENT VALUES & CHOICES<<

1. FUNCTION
F(X)=SIN(X)

2. LIMITS
X: [-1,7]
Y: [-1.2,1.2]
SCALING: MANUAL

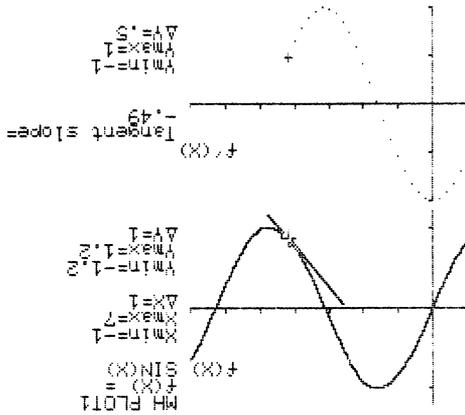
ACTIONS:

6. SHOW CURRENT DISPLAY
7. PLOT
8. EXIT

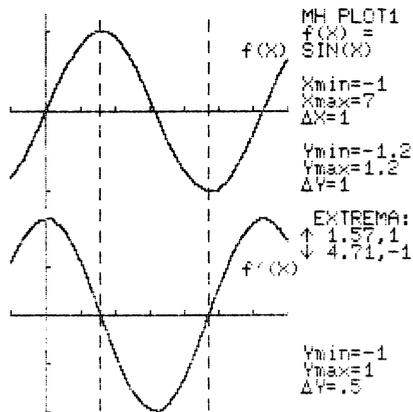
CHOICE: CHANGE VALUE(1,2)
ACT(6-8) WHICH?

Press "7" for the built-in example. The original function F(X) is graphed at the upper left of the screen, with scaling done manually or automatically as desired. Descriptive data is listed on the right.

When you follow the instruction to "PRESS ANY KEY TO START/STOP", the graph is retraced with some embellishments. A "+" is placed at the current point, and the tangent line at that point is drawn. The slope of the tangent line is plotted as a single point on the graph of $f'(x)$ at the bottom of the screen (preceded by a flashing "+" at the bottom of the screen to show what is about to happen). Then the tangent line disappears, the next point to the right is selected on the upper graph, and the process is repeated. You can stop the action at any time by pressing any key. The tangent slope will then appear on the right as "TANGENT SLOPE ="



When you follow the instruction to "PRESS ANY KEY TO



The next phase is the test for extrema. The program first looks for changes of sign of $F'(X)$. For each change of sign, some "zeroing in" is done (by a secant method) to find the corresponding zero of F' . A vertical line is drawn through this value of X , intersecting the graph of $F(X)$ at an extremum. This point is then classified as a local maximum or minimum and listed on the right side of the screen. The procedure is repeated until all sign changes are exhausted. (If no extrema are found, the message "EXTREMA: NONE" is flashed.)

In addition to the extrema, information about scaling is listed at the right. The values X_{\min} and X_{\max} give the extremes of X on both graphs, ΔX is the distance between successive tic-marks on the X -axis. Likewise, Y_{\min} , Y_{\max} , and ΔY are given for each of the two graphs.

In some applications of MH PLOT1, the intermediate displays are more interesting than the final one. If you wish to save an intermediate display, freeze the image during the

plotting of $F'(X)$ by pressing any key, then press Ctrl-E to exit. This will take you to the program's menu, from which you can exit to the main ARBPLOT menu. Choosing #9, SAVE DISPLAY, will allow you to capture the image you previously froze.

Warning: MH PLOT1 uses numerical differentiation to obtain $F'(X)$ and can be fooled. If the function deviates wildly on the domain, unpredictable results can occur. For example, the function $TAN(X)$ has many vertical asymptotes on the domain $[0,100]$, and not enough values of $F(X)$ will be sampled to do justice to the computation of $F'(X)$. This can usually be remedied by suitably restricting the function's domain; in the case of $TAN(X)$, the domain $[-1.5,1.5]$ does well.

3.2.3. Roots of $F(X)=0$

ARBROOT finds roots of $F(X)=0$. There are two steps to finding a real root of a real equation. First, the root must be isolated, i.e., the root must be shown to lie in a known interval. If the function is continuous, it suffices for the function to change sign in the interval. Second, the isolated root must be evaluated to the desired degree of accuracy. ARBROOT solves the second problem for continuous functions. If the function is discontinuous, peculiar things can happen.

To use ARBROOT, select #7 in the main ARBPLOT menu.

>>ARBROOT - CURRENT VALUES & CHOICES<<

1. FUNCTION
F(X)=X²-5
F'(X)=2*X
2. BOUNDS ON ROOTS
XMIN=0 XMAX=5
3. METHOD: NEWTON
4. SCREEN: CLEAR PRESENT DISPLAY

ACTIONS:

6. SHOW CURRENT DISPLAY
7. FIND ROOT
8. EXIT

CHOICE: CHANGE VALUE(1-4)
ACT(6-8)

WHICH(1-8)?

As with other programs, you must specify the function to be plotted and limits on the independent variable. You do not specify DELTA X; 49 points are automatically plotted. ARBROOT finds roots by three methods. Choice 3 in the ARBROOT menu gives the subsidiary menu for selecting the method.

- - METHOD - -

PRESENT CHOICE IS NEWTON

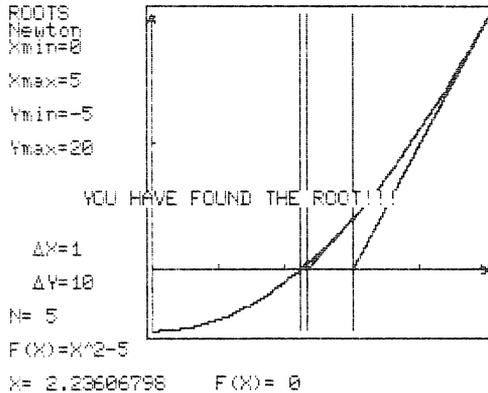
THE CHOICES ARE:

1. BISECTION
2. REGULA FALSI
3. NEWTON

WHICH(1-3)?

Before exploring other methods, press "7" in the ARBROOT menu to see Newton's Method of finding the square root of 5. The program plots the curve, draws the first tangent line, and obtains the first approximation. Press "C" to get the next

approximation and then continue pressing "C" until the message "YOU HAVE FOUND THE ROOT!!!" appears. Note that each time you press "C", new values for N (the iteration count), X (the current approximation), and F(X) (the value of the function) appear on the screen along with the depiction of the action.



How did the program know the root had been found? Any iterative procedure must have a criterion for convergence. ARBROOT has two. The root is considered found when either two successive approximations differ by less than $.5 \times 10^{-6}$ or the absolute value of the function is less than 10^{-8} .

Now return to the ARBROOT menu by pressing "E". Change the method to Bisection and press "7" to plot. Then press "C" a few times to observe the process. Bisection inevitably but slowly converges. Along the way, one approximation may move farther from the root than its predecessor. Each approximation cuts the interval in half; soon you cannot distinguish two successive approximations. Now press "Z" to zoom. Function values are recomputed and a new plot is drawn with the previous

smallest interval expanded to fill the screen. Continue with "C" and "Z" until you reach the root.

Return to the ARBROOT menu again by pressing "E". Change the method to Regula Falsi, press "7", and plot again. Press "C" a few times to observe the process. Zooming in Regula Falsi or Newton is usually not very effective, since one end of the interval usually remains fixed.

Newton's Method usually converges fastest, but it is not foolproof. To illustrate, set $F(X)=\cos(X)$, return to Newton's Method, setting $F'(X)=-\sin(X)$, and press "7". First, you will get an error message since $\cos(0)$ and $\cos(5)$ are both positive. ARBROOT needs opposite signs. Change the interval to $0 \leq X \leq 4$ and try again. Press "7" to start. There is a root in the interval, but Newton's Method would miss it. The tangent from the right end of the interval would lead outside the interval; the tangent from the left end is horizontal. ARBROOT forces convergence. First, it calculates the tangent line at the right end of the interval. If that line fails to cross the X-axis within the interval, the tangent line at the left end is calculated. If that also fails, ARBROOT turns to bisection. Continue pressing "C" until the root is found. One bisection suffices to put Newton's Method on the way to convergence.

Try one more example, still with Newton's Method. Set $F(X)=X^3-X+1$, $F'(X)=3X^2-1$, $X_{\min}=-2$, $X_{\max}=2$, then plot. Note what happens when $N=4$.

3.3. Sequences and Series

There is a single program giving graphic treatment of

sequences and series, ARBSEQ, selection #8, on ARBPLOT Disk 1.

ARBSEQ presents the terms of a sequence, and optionally, the partial sums of the associated series and a function of the terms of the sequence. The sequence can be defined explicitly or recursively.

In the main ARBPLOT menu, select #8, SEQUENCES & SERIES.

>>ARBSEQ - CURRENT VALUES & CHOICES<<

1. FUNCTIONS

SEQ: $U(N) = 2 * U(N-1) / N$

FCN: $F(N) = U(N) / U(N-1)$

2. LIMITS AND SPECIAL VALUES

N= 0 TO 20

U-VALUES:1,

3. SCREEN: CLEAR PRESENT DISPLAY

4. DISPLAY SUMS: YES

5. FUNCTION LIMITS:

LIM U(N) = 0

LIM S(N) = 7.389

LIM F(N) = NONE

ACTIONS:

6. SHOW CURRENT DISPLAY

7. PLOT SERIES

8. EXIT

CHOICE: CHANGE VALUE(1-5)

ACT(6-8)

WHICH(1-8)?

Before plotting, let's examine the menu carefully; ARBSEQ differs in several ways from the other programs of ARBPLOT.

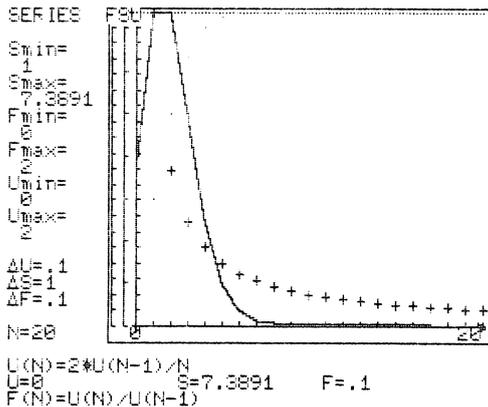
First, look at the function definition. The terms of the sequence are $U(N)$. The definition may be explicit or recursive. The example given is the recursive definition of $2^N/N!$ (It isn't possible to define $N!$ explicitly in BASIC). You can also define an auxiliary function of the elements or explicit definition of another sequence. The example is an auxiliary function: the ratio of successive terms of the sequence.

Next, you state the beginning and ending subscripts for the sequence. Once started, you can answer "C" to continue a sequence. If you have defined the sequence recursively, you must give enough special values to get started. You may specify initial values even if the sequence definition does not require them.

You may clear or keep the previous display if you wish to compare two sequences. You may calculate and display sums or not, as you see fit.

Although ARBSEQ can only calculate and display a finite number of terms, you may enter a limit for the sequence, sum, or function. The built-in example starts with limits given for terms of the sequence, the sum(e^2), but not the function.

Now press "7" to plot. Notice that you can pause by pressing any key, then continue by pressing again or exit with Ctrl-E. An odometer runs on N, U, S, and F. Independent vertical scales are given for the three dependent variables. U(N) is plotted as a solid line, S(N) as a dotted line, and F(N) as "+"s.



Return to the menu to try another example. Set $U(N)=U(N-1)+U(N-2)$ (Fibonacci sequence), $F(N)=U(N)/U(N-1)$ (Ratio of successive terms) for $N=1$ to 10. You must give the values for $U(1)=1$ and $U(2)=1$, since $U(N)$ uses $U(N-1)$ and $U(N-2)$. Set $LIM F(N)=1.618$, do not display sums, and then plot. Notice the pattern of convergence of the ratio. Return to the menu and continue for 20 more terms by pressing "2", then answering "C" and "20" before plotting again.

3.4. Analytic Geometry: Conic Sections

The first two programs on Disk 2 treat the analytic geometry of quadratic equations in two dimensions, the conic sections. As shown on the second main ARBPLOT menu, these are

1. CONIC SECTIONS (Program CONIC)
2. ROTATION OF AXES; ELLIPSE (Program MH PLOT4)

CONIC is a comprehensive treatment of the algebra and geometry of ellipses, hyperbolas, and parabolas whose axes are parallel to the coordinate axes. MH PLOT4 is a demonstration of the relationship between the coefficients of a quadratic and the angle between the coordinate axes and axes of an ellipse.

These two programs differ significantly from the other programs in ARBPLOT in that they explore a specific family of equations, the quadratics and their relatives in polar coordinates, rather than arbitrary functions.

3.4.1. Conic Sections

CONIC explores many aspects of the algebra and geometry of quadratic equations. In the second main ARBPLOT menu (Disk 2), select #1, CONIC SECTIONS.

>>CONIC - CURRENT VALUES & CHOICES<<

1. EQUATION: ELLIPSE
 $4X^2 + 9Y^2 - 8X + 18Y - 23 = 0$

2. LIMITS/SCALING
XMIN=-2 XMAX=4
YMIN=-4 YMAX=2
SCALING: AUTOMATIC

3. PROBLEM: DRAW

4. SCREEN: CLEAR PRESENT DISPLAY
5. AXES: SHOW

ACTIONS:
6. SHOW CURRENT DISPLAY
7. PLOT CURVE
8. EXIT

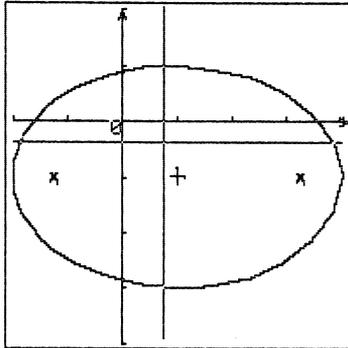
CHOICE: CHANGE VALUE(1-5)
ACT(6-8) WHICH(1-8)?

Before exploring the choices, press "7" to plot. An odometer on "I" runs from zero to 64; an angle rotates through one revolution in 64ths of a circle about one focus to draw the curve. At the end, the foci are labelled with "x" and the center (or vertex for a parabola) with "+". Press any key to return to the menu.

CONIC
Ellipse

Xmin=-2
Xmax=4
Ymin=-2.99
Ymax=.995

$\Delta X=1$
 $\Delta Y=1$
I = 64



$4X^2 + 9Y^2 - 2X + 11.3Y - 32.2 = 0$
(.75, -.375)

The equation may be expressed in three ways, selected from a subsidiary menu by pressing "1".

- EQUATION -

PRESENT TYPE IS GENERAL QUADRATIC

EQUATION TYPES ARE:

1. GENERAL QUADRATIC
2. STANDARD QUADRATIC
3. POLAR

WHICH(1-3)?

Reject GENERAL QUADRATIC (press "N") and select STANDARD QUADRATIC. You see the same ellipse described in standard form; accept this (press "Y"). We shall return to POLAR later.

At the CONIC menu, press "3". Reject DRAW in order to see the PROBLEM menu:

- PROBLEM -

PRESENT CHOICE IS DRAW

PROBLEM CHOICES:

1. DRAW
2. TRANSLATE AXES
3. SUM OR DIFFERENCE
4. ECCENTRICITY
5. ASYMPTOTES

WHICH(1-5)?

Now select TRANSLATE AXES, return to the CONIC menu, and plot. When the plot is completed, "L,R,U,D,E?" appears, meaning "move axes left, right, up, down, or exit." Press "R" to move the Y-axis to the right. The coordinates of the new axes relative to the old are given in the lower right corner, and the equation is shown in the new coordinate system. Now press "R" and "D" until the equation has the form $((X)/3)^2 + ((Y)/2)^2 = 1$ with new origin at (1,-1) relative to the original. Press "E" to return to the CONIC menu; your new equation is now the program equation. Press "7" to plot again. The curve is in the same location on the screen, but the axes are in the center.

With this introduction, let's now examine the options of CONIC in detail.

The EQUATION can be defined in three ways: 1) as the general quadratic

$$ax^2 + cy^2 + dx + ey + f = 0$$

2) as the standard quadratic

$$\left(\frac{x-h}{a}\right)^2 \pm \left(\frac{y-k}{b}\right)^2 = \pm 1$$

(for circles, ellipses, or hyperbolas), or

$$(x-h)^2 = \pm 2p(y-k) \quad \text{or} \quad (y-k)^2 = \pm 2p(x-h)$$

(for parabolas); or 3) in standard polar form with focus at the origin

$$r = \frac{ep}{1 \pm e \cos t} \quad \text{or} \quad r = \frac{ep}{1 \pm e \sin t}$$

where e is eccentricity and p is the distance from focus to directrix.

Defining the function in one form automatically calculates the coefficients of the same curve in the same position in the other forms, with the exception that the polar form is always in standard position. The coefficients are printed to three significant figures, but eight significant figures are carried internally.

Change PROBLEM back to DRAW, change EQUATION to POLAR, accept the equation, plot it. Now change EQUATION back to GENERAL QUADRATIC; the X-term has reappeared, since the origin has moved from the center to a focus.

LIMITS/SCALING can be AUTOMATIC or MANUAL. Units on the two axes are the same unless you deliberately change them with MANUAL scaling. With AUTOMATIC scaling, central conics have the center in the middle of the screen, and parabolas have the focus in the middle. The size is appropriate to the problem being solved.

Five PROBLEMS can be solved. DRAW simply plots the curve. TRANSLATE AXES, as you have seen, shows the relation between the location of the axes and the coefficients of the quadratic equation. Since translation doesn't apply to the standard polar form, a polar equation will automatically be converted to standard quadratic form if TRANSLATE AXES has been chosen. The increment in moving the axes is $1/16$ the width of the screen, jumping to (H,K) when within one-half unit of that point.

SUM OR DIFFERENCE lets you explore one way to define circles, ellipses, and hyperbolas. You may either start with an

equation and show the sum or difference geometry, or you may specify the distance between two fixed points, the sum or difference of distances from a variable point to those points, and generate both equation and geometry.

ECCENTRICITY lets you explore the definition of an ellipse, hyperbola, or parabola in terms of the ratio between distances of a variable point from a fixed point and fixed line. ECCENTRICITY shows the geometry. You can specify the relationships with the equation in polar form, or you can observe the geometry for any of these shapes with the equation given in quadratic form.

ASYMPTOTES draws the asymptotes of a hyperbola before plotting.

Trying to specify a problem which is not applicable to a curve, e.g., show the asymptotes of an ellipse, will cause the problem to be rejected. Changing the curve to make a previously valid problem choice invalid will change the problem to the nearest valid form.

The default SCREEN setting is to CLEAR PRESENT DISPLAY. Changing to KEEP automatically changes the scaling to manual, and gives you the opportunity to change the limits if you wish.

The default AXES setting is to SHOW them. When you are exploring the geometry of SUM OR DIFFERENCES or ECCENTRICITY, it is sometimes helpful to DON'T SHOW the axes in order to emphasize the geometric relationships.

Once you have set all parameters, PLOT CURVE plots the curve. ASYMPTOTES, if chosen, are drawn first. The lines associated with SUM OR DIFFERENCE or ECCENTRICITY are drawn in

animation while the curve is plotted. TRANSLATE AXES is done after the curve is completed.

The way the lines for SUM OR DIFFERENCE and ECCENTRICITY are drawn may cause portions of the curve to be wiped out while being plotted. Therefore, in these two cases the curve is automatically redrawn a second time without the extra lines. In KEEP mode, a previously drawn curve may in these two cases be decimated. Care must be exercised in planning demonstrations involving several related curves. For example, when using SUM OR DIFFERENCE to draw a family of ellipses with the same focal distance, work from the outside in rather than inside out to avoid erasing parts of a previously drawn curve.

3.4.2. Rotation of Axes; Ellipse

MH PLOT4 demonstrates the effect of rotation of axes on the equation of an ellipse. The program describes the change of variables associated with a rotation of axes and discusses the application to quadratic equations and conic sections. The program then uses the graph of the ellipse given by

$$7x^2 - 6\sqrt{3}xy + 13y^2 = 16$$

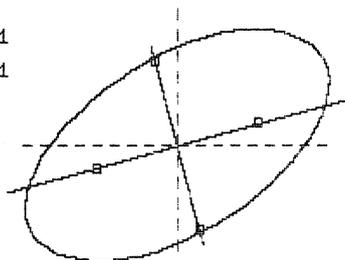
to illustrate rotation of axes.

In the second main ARBPLOT menu, select #2, ROTATION OF AXES; ELLIPSE. The program starts with a tutorial on axis rotation. You can skip through this quickly if you only want the demonstration, but it may be desirable to use the text in the classroom to present rotation.

←,→,E?

$$\Delta X = 1$$

$$\Delta Y = 1$$



ROT = 15

$$4.8x^2 - 6xy + 15.2y^2 = 16$$

After some initial calibration (see below), you control the rotation of the coordinate axes using the "→" and "←" keys. Each depression of one of these keys brings about a five degree rotation of the axes ("←" for counter-clockwise, "→" for clockwise), and for each resultant orientation the equation in terms of the new variables is displayed on the screen. You can rotate counter-clockwise from 0 to 90 degrees. At a rotation of 30 degrees, the equation reduces to $4x^2 + 16y^2 = 16$. The elimination of the xy term is accomplished, and the lengths and inclination of the major and minor axes are evident. Further rotations obscure these features again and bring back the xy term. At any time, you can press "E" to exit; you will then be given the chance to restart the program or exit to the main ARBPLOT menu.

In the usual introduction to rotation of axes, many students get the impression that they are under some obligation to rotate through a specific angle. This program is intended to emphasize the fact that rotation is arbitrary, but that the

utility of the resulting equation depends on the choice of rotation.

Calibration. Rotated coordinate axes may appear perpendicular on one video screen and not another since the height to width ratio is not constant from set to set. The calibration phase of the program is required to adapt the graphing to your screen. Two lines will be displayed which may or may not appear perpendicular. The cursor keys "→" and "←" will change the angle between the lines; when this is a right angle, press "E" to get on with the program.

3.5. Programs Involving Integration

Five programs on Disk 2 do integration. As shown on the second main ARBPLOT menu, these are

- | | |
|----------------------------|--------------------|
| 3. DEFINITE INTEGRAL | (Program ARBINT5) |
| 4. INDEFINITE INTEGRAL | (Program MH PLOT2) |
| 5. MONTE CARLO INTEGRATION | (Program ARBINT1) |
| 6. POLAR INTEGRATION | (Program ARBINT6) |
| 7. ARC LENGTH | (Program ARBINT7) |

ARBINT5 does numerical integration by rectangles (Fundamental Theorem), trapezoids, and parabolas (Simpson's Rule). MH PLOT2 illustrates the geometric relationship between a function and its indefinite integral. ARBINT1 approximates the area under a curve by a Monte Carlo dart-throwing process. ARBINT6 shows the Fundamental Theorem for integration in polar coordinates. ARBINT7 shows the fundamental theorem for integration to determine arc length in rectangular, parametric

rectangular, and polar coordinates.

All of the programs allow you to use any function bounded and piecewise continuous throughout any finite interval you specify. All use automatic scaling and automatic determination of the plotting increment, then plot the curve(s).

After plotting, the program switches to integration mode. For all but the indefinite integral, the integral being evaluated appears at the bottom of the screen, then the program carries out its particular integration process. At the end, the antiderivative (if any) is evaluated, and its value to six significant figures is reported beneath the integral with the word "(EXACT)".

3.5.1. Definite Integral

ARBINT5 gives a graphic presentation of three methods of numerical integration of the difference of two functions. It can treat the methods separately or together. In addition, it can show a typical element for integration, and it can perform Richardson Extrapolation from Trapezoidal Rule integration. In the second main ARBPLOT menu, select #3, DEFINITE INTEGRAL.

>>ARBINT5 - CURRENT VALUES & CHOICES<<

1. FUNCTIONS

$$Y1(X) = 1 / (1 + X^2)$$

$$Y2(X) = 0$$

2. LIMITS

$$XMIN = -1$$

$$XMAX = 4$$

3. NUMBER OF SUBDIVISIONS: 4

4. METHOD: ALL THREE

5. ANTIDERIVATIVE:

$$I(X) = \text{ATN}(X)$$

ACTIONS:

6. SHOW CURRENT DISPLAY

7. INTEGRATE

8. EXIT

CHOICE: CHANGE VALUE(1-5)

ACT(6-8)

WHICH(1-8)?

The integration method to be used is selected through Choice 4, which leads to a subsidiary menu.

- - METHOD - -

PRESENT IS ALL THREE

THE CHOICES ARE:

1. RECTANGLES (FUNDAMENTAL TH.)

2. TRAPEZOIDS

3. PARABOLAS (SIMPSON'S RULE)

4. ALL THREE

5. TYPICAL ELEMENT

6. RICHARDSON EXTRAPOLATION

WHICH(1-6)?

Return to the ARBINT5 menu by answering "4" and press "7" to run the built-in example.

INTEGRATION
Three Rules

Xmin=-1
Xmax=4
Ymin=0
Ymax=.998

R=2.33207

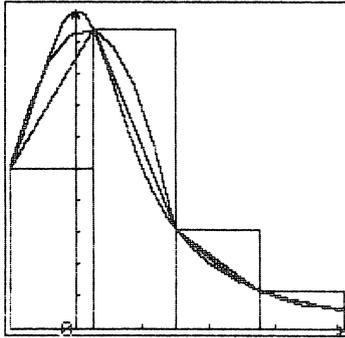
T=2.05634

S=2.25253

$\Delta X=1$
 $\Delta Y=.1$

N = 4

$$I = \int_{-1}^4 \frac{1}{(1+X^2)} dX = 2.11122 \quad (\text{EXACT})$$



After plotting the curves, the integration process begins. ALL THREE calculates and displays rectangles, trapezoids, and parabolas. The rectangles are based at the left edge of each subdivision. The rectangles and trapezoids are calculated and displayed simultaneously, then the parabolic strips for Simpson's Rule are calculated and displayed. You may pause by pressing any key in the midst of drawing rectangles and trapezoids, but not while drawing parabolas, then continue by again pressing a key. After all plotting is completed, sums are displayed on the left, "R" for rectangles, "T" for trapezoids, and "S" for Simpson's Rule. The number of subdivisions, "N", also appears. Pressing any key returns you to the ARBINT5 menu; pressing "6" then recalls the display if desired.

Press "3" to change the number of subdivisions. Try an odd number. Your choice will be rejected since ALL THREE includes Simpson's Rule. Now try a larger even number, then press "7" to see the change in the display and calculation.

The maximum N for an effective display is 64, although the process can be carried out with up to 200 subdivisions.

You can change the limits of plotting and integrating by pressing "2". Remember that your function must be defined throughout the interval.

Now let's return to choosing the method. Press "4" to get the METHOD menu. Select #1, RECTANGLES (FUNDAMENTAL TH.), return to the ARBINT5 menu, and integrate. Two rectangles appear on each subdivision, one based on the left edge and one on the right. Three sums appear, "L" for the left rectangles, "M" for rectangles based at the midpoint, and "R" at the right.

Return to the menu to try another method. Select #2, TRAPEZOIDS. Only the trapezoids are drawn and summed. Similarly, only parabolic strips are drawn and summed with selection #3, PARABOLAS (SIMPSON'S RULE).

TYPICAL ELEMENT, selection #5 in the METHOD menu, does no summing. Instead, it shows a typical rectangular element for the area. You must pick the location of the element, between Xmin and Xmax.

Finally, selection #6, RICHARDSON EXTRAPOLATION, does an "extrapolation to zero grid size" based on the Trapezoidal Rule. The mathematics of Richardson Extrapolation is given in Appendix B. Press "7" to integrate. Unless you have just done the Trapezoidal Rule, selection #2 or #4 in the METHOD menu, you will get the message "RUN TRAPEZOIDS BEFORE RICHARDSON". Run the Trapezoidal Rule, then Richardson. The display will return with an additional line on the left, "RIC=" the improved value.

You can change the function and integrate to find the area between two curves. Press "1" at the ARBINT5 menu to change the functions. Set $Y1(X)=X-X^2$, $Y2(X)=X^3-X$. Change the interval to $Xmin=0$, $Xmax=1$, and set the method to ALL THREE. Integrate; observe that "NO ANTIDERIVATIVE GIVEN" appears at the bottom of the screen. Changing the function wipes out the previous antiderivative.

Return to the ARBINT5 menu and press "5". You will be asked to enter the antiderivative of $Y1(X)-Y2(X)$ and will be given $Y1-Y2=X-X^2-(X^3-X)$. Set $I(X)=-\frac{X^3}{3}+X^4/4+X^2$. Remember the care you must take with leading minus signs. As soon as you have accepted your choice of antiderivative, the display returns with the exact value of the integral added.

3.5.2. Indefinite Integral

MH PLOT2 illustrates the geometric relation between a function and its indefinite integral by dynamically shading areas. The program also offers a quick way to evaluate and interpret a definite integral. The only input required is the function $F(X)$ (which must be bounded and piecewise continuous) and the interval on which it is to be integrated (scaling is automatic). The program displays $F(X)$ at the top of the screen, and plots the integral at the bottom by "accumulating" area in the original graph and plotting corresponding points. The integral is computed numerically by Simpson's Rule.

To see how the program works, choose #4, INDEFINITE INTEGRAL, from the main ARBPLOT menu on Disk 2.

>>MH PLOT2 - CURRENT VALUES & CHOICES<<

1. FUNCTION

$$F(X)=EXP(-X)$$

2. LIMITS

$$XMIN = 0$$

$$XMAX = 4$$

ACTIONS:

6. SHOW CURRENT DISPLAY

7. PLOT

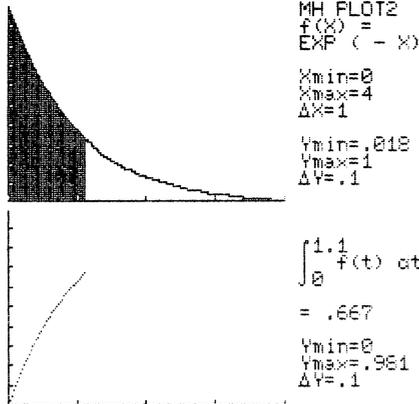
8. EXIT

CHOICE: CHANGE VALUE(1,2)

ACT(6-8)

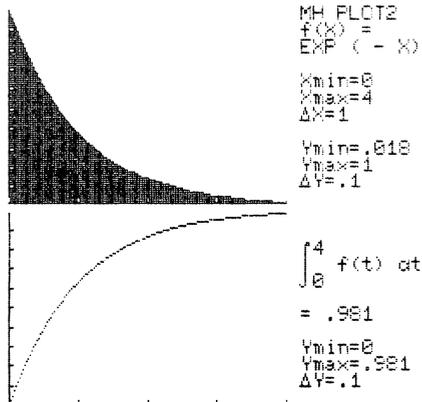
WHICH? ^

Leave the function $F(X)=EXP(-X)$ and its domain $[0,3]$ alone and select #7, PLOT. The program will calculate for a bit, then graph $F(X)$. The program will then inform you that you should press any key to start/stop. This means that you must press a key for action to begin and that depressing a subsequent key will freeze the picture during the graphing of the integral (a third keystroke will resume the action). After the program gets the go-ahead, it begins sweeping and shading the area under $F(X)$ and plotting the integral. If you freeze the action during the plot you will see something like the following figure on the screen:



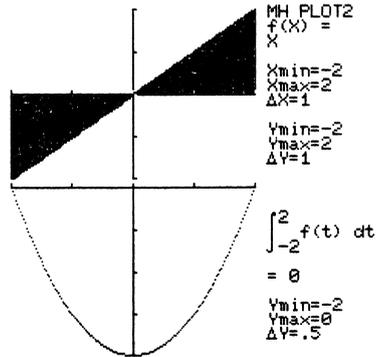
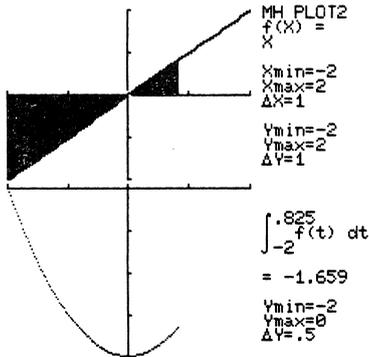
Note that the integral from the domain's left endpoint to the current value of X is given on the right of the screen. This feature can be used, for example, to have a class guess when the integral will attain a given value, say 0, to show cancellation of signed areas.

If you hit Ctrl-E while the picture is frozen, the program halts the plot and returns to the menu. You can then exit the program and use SAVE DISPLAY to capture the picture as it was frozen. If you allow the plot to finish, the screen should look like this:



On the right of the screen, the X and Y limits for each graph are given, as well as the distance between successive tic-marks. Also, the value of the integral from Xmin to Xmax is given.

Press any key to return to the menu and try a function of your own. Choose option 1 to change the function, and enter $F(X)=X$. Use option 2 to change the domain to $[-2,2]$, then from the menu press "7" to plot. Freeze the action as desired and return to the menu when you are finished. The intermediate and final graphs should look like those displayed in the figures below. Note that the graph can be displayed again immediately through option 6. Choose option 8 to return to the main ARB-PLOT menu, from which you can save the display, run another program, or quit.



3.5.3. Monte Carlo Integration

A different way of thinking about the area under a curve is to think of the rectangle enclosing the curve as a dart-board. If dart hits on the board are uniformly distributed from left to right and top to bottom, then the area under the curve is approximated by the area of the rectangle times the ratio of hits (darts landing in the area under the curve) to total darts thrown. ARBINT1 shows this process. In the second ARBPLOT main menu (Disk 2), select #5, MONTE CARLO INTEGRATION.

>>ARBINT1 - CURRENT VALUES & CHOICES<<

1. FUNCTION
 $Y(X) = 1 / (1 + X^2)$
2. LIMITS
XMIN=-1 XMAX=4
3. NUMBER OF DARTS: 100
4. ANTIDERIVATIVE:
 $I(X) = \text{ATN}(X)$

ACTIONS:

6. SHOW CURRENT DISPLAY
7. INTEGRATE
8. EXIT

CHOICE: CHANGE VALUE(1-4)
ACT(6-8)

WHICH (1-8)?

Now press "7" to run the program.

INTEGRATION
Monte Carlo

Xmin=-1
Xmax=4

Ymin=0
Ymax=.998

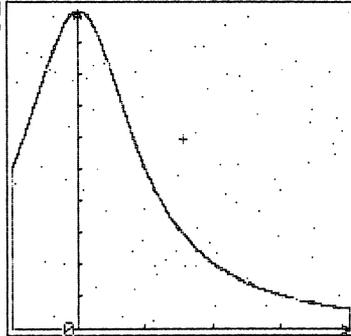
DARTS=80

HITS=32

I=1.99653

$\Delta X=1$

$\Delta Y=.1$



$$I = \int_{-1}^4 \frac{1}{(1+x^2)} dx = 2.11122 \quad (\text{EXACT})$$

You see a flash as each dart strikes, with an update of DARTS, HITS, and I (for integral). You may pause during the run by pressing any key, then continue by pressing the key a second time.

When the run is completed, press any key to return to the ARBINT1 menu. You can then change the function; it must, however, be positive. An error message will appear at CALCULATE time if the function value goes negative at any point in the interval. You may, but need not, provide an antiderivative. ARBINT1 checks to see that your antiderivative is a legal function, but not that it is the correct antiderivative.

Once you have defined a function and limits and "thrown" some darts, you can throw still more. At the ARBINT1 menu, press "3". You can continue ("C") to build a bigger sample, or you can start over with a new sample. In any Monte Carlo process, including this one, it is easy to get one significant figure in a result, reasonable to try for two significant

figures, and very difficult to get more than two.

3.5.4. Polar Integration

ARBINT6 presents the fundamental theorem for integration of the difference of two functions in polar coordinates. It shows elements of integration as circular sectors with radius the value of R at the midpoint of each interval, and permits showing either all sectors and their sum or a typical element.

In the second main ARBPLOT menu (Disk 2), select #6, POLAR INTEGRATION.

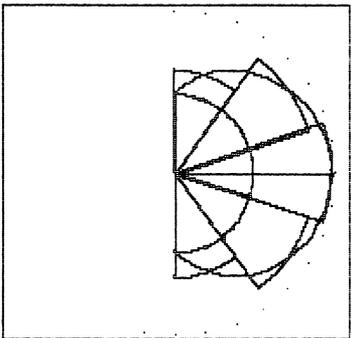
```
>>ARBINT6 - CURRENT VALUES & CHOICES<<
1. FUNCTIONS
  R1(T)=1+COS(T)
  R2(T)=1
2. LIMITS (T IN RADIANS)
  TMIN=-1.57      TMAX=1.57
3. NUMBER OF SUBDIVISIONS:5
4. MODE: SUM
5. ANTIDERIVATIVE:
  I(T)= SIN(2*T)/8+SIN(T)+T/4
ACTIONS:
6. SHOW CURRENT DISPLAY
7. INTEGRATE
8. EXIT
CHOICE: CHANGE VALUE(1-5)
        ACT(6-8)
        WHICH(1-8)?
```

Now press #7 to run.

```

INTEGRATION
Polar
Tmin=-1.57
Tmax=1.57
Rmin=1
Rmax=2
L=2.71916
M=2.81868
R=2.71916
  ΔR=1
  N= 5

```



$$I = \int_{-1.57}^{1.57} .5 * ((1 + \cos(T))^2 - (1)^2) dT \quad (\text{EXACT}) = 2.7854$$

The program first plots the curves and then switches to integration mode to draw the sectors. You may pause while the sectors are being drawn by pressing any key, then press any key again to resume the action. Although only the sectors based at the midpoint of each interval are plotted, the sectors based at the left and right edges are also calculated, summed, and reported on the left side of the screen as L, M, and R.

Return to the ARBINT6 menu by pressing any key. You can show a typical element by pressing "4" which automatically toggles between SUM and ELEMENT. When you switch to ELEMENT, you must give the location. Set the location to T=.6, then press "7" to plot again. The element must lie between Tmin and Tmax.

In polar coordinates, area within a curve is always positive, even if R(T) is negative, but we can get a negative integral if R1 is less than R2. Press "1" to change the function. Set R1(T)=1, R2(T)=1+cos(T), then press "7" to integrate, and note the result.

Return to the ARBINT6 menu. Notice that changing the functions changed the antiderivative to NONE. Change the functions again, this time to $R1(T)=\sin(2*T)$, $R2(T)=0$, then press "5" to specify the antiderivative. You get the message

```
ENTER ANTIDERIV OF (R1(T)^2-R2(T)^2)/2
(R1^2-R2^2)/2=.5*((SIN(2*T))^2)
I(T)=.....
```

Enter your derivative, in this case

```
I(T)=.25*T-.125*SIN(4*T).
```

The program checks to see that you have a legal function, but not that it is the correct one.

Change the interval to $T_{min}=0$, $T_{max}=5.5$, and the number of subdivisions to 12, then integrate. Note the straight line at $T=5.5$, marking the end of the interval. Observe that the area of the sectors is positive, even though the radius is negative. Try showing a typical element for $T=1$ and for $T=2$.

Finally, how can we obtain the area between two loops of the same curve? Given the logarithmic spiral, $R(T)=\exp(.1*T)$, find the area between the portion of the curve with $0 \leq T \leq 2\pi$ and that with $2\pi \leq T \leq 4\pi$. Try setting $R1(T)=\exp(.1*(T-6.28))$, $R2(T)=\exp(.1*T)$, $T_{min}=0$, $T_{max}=6.28$, and integrate.

3.5.5. Arc Length

ARBINT7 presents the fundamental theorem for integration to find arc length in rectangular, parametric rectangular, or polar coordinates. It can show either the sum of chords or a typical element. In the second ARBPLOT main menu (Disk 2),

select #7, ARC LENGTH.

>>ARBINT7 - CURRENT VALUES & CHOICES<<

1. FUNCTION: POLAR
 $R(T) = \text{EXP}(.2 * T)$
 2. LIMITS(T IN RADIANS)
TMIN=0 TMAX=6.28
 3. NUMBER OF SUBDIVISIONS: 5
 4. MODE: SUM
 5. ANTIDERIVATIVE:
 $I(T) = 5.099 * \text{EXP}(.2 * T)$
- ACTIONS:
6. SHOW CURRENT DISPLAY
7. INTEGRATE
8. EXIT

CHOICE: CHANGE VALUE(1-5)
ACT(6-8) WHICH(1-8)?

Select the coordinate system by pressing "1" for FUNCTION.

Reject POLAR, press "N", and you will get the subsidiary menu:

- - FUNCTION - -

PRESENT CHOICE IS POLAR

- THE CHOICES ARE:
1. RECTANGULAR
 2. PARAMETRIC
 3. POLAR

WHICH (1-3)?

Now select POLAR ("3"), accept the function ("Y"), and integrate by pressing "7" at the ARBINT7 menu.

INTEGRATION
Arc Length

Tmin=0
Tmax=6.28

Xmin=-4.91
Xmax=3.51

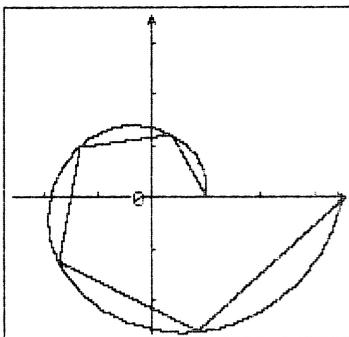
Ymin=-2.62
Ymax=1.4

SUM=11.9828

ΔX=1

ΔY=1

N= 5



$$I = \int_0^{6.28} \text{SQR}(R^2 + (R')^2) dT = 12.865 \quad (\text{EXACT})$$

The program first draws the Spira Mirabilis or logarithmic spiral, then switches to integration mode to draw and sum the chords. The sum and number of chords appear on the left side of the screen. Return to the ARBINT7 menu and press "4" to show a typical element. Pick a suitable location between Tmin and Tmax, say T=4, then press "7" to see the changed display.

Return again to the ARBINT7 menu and change the function. This time try PARAMETRIC, with X(T)=COS(T), Y(T)=SIN(T). Note that changing the function eliminates the antiderivative. Press "5" to supply an antiderivative. For these parametric equations, you get:

- - ANTIDERIVATIVE - -

PRESENT CHOICE IS NONE

FIRST YOU NEED X'(T) & Y'(T).

X(T)=COS(T)

Y(T)=SIN(T)

X'(T)=-SIN(T).....

Y'(T)=COS(T).....

NOW ENTER THE ANTIDERIVATIVE OF

SQR((X')^2+(Y')^2)

I(T) =

You supply the derivatives of $X(T)$ and $Y(T)$. The program does not use them; you supply them as the first step toward the antiderivative. The program now gives you the model, and you find and enter the antiderivative.

Leave the limits and number of subdivisions unchanged, press "4" to switch back to SUM, and press "7" to integrate.

Whichever coordinate system you have chosen, you will be asked for a function in the proper form. You will then be prompted for the necessary ingredients and model for the antiderivative, should you choose to enter one. Try an example of your own making in rectangular coordinates, $Y(X)$.

Take another look at the display; press "6" if you are not already at the display. Observe that for arc length, and for arc length only, the display does not show the specific function being integrated, only the general form. The usual length of the integrand for arc length would overflow onto the next line and be unreadable.

3.6. Direction Fields and Differential Equations

MH PLOT3 plots solutions and/or direction fields of first-order differential equations of the form $DY/DX=F(X,Y)$. If a solution (integral curve) is desired, an initial condition $Y(X_0)=Y_0$ must be specified. The direction field option shows tangent lines determined by the differential equation on a representative grid of points. (See Appendix A for a brief description of direction fields.)

Enter MH PLOT3 by selecting #8, DIFF. EQUATIONS/
DIRECTION FIELDS, from the ARBPLOT menu on Disk 2.

>>MH PLOT3 - CURRENT VALUES & CHOICES<<

1. DIFFERENTIAL EQUATION
DY/DX=X-Y

2. INITIAL CONDITION: Y(0)=1

3. MODE:
DIRECTION FIELD AND INTEGRAL CURVE

4. LIMITS
X: [-1,8]
Y: [-1,8]
SCALING: MANUAL

5. SCREEN: CLEAR PRESENT DISPLAY

ACTIONS:

6. SHOW CURRENT DISPLAY

7. GRAPH

8. EXIT

CHOICE: CHANGE VALUE(1-5)

ACT(6-8)

WHICH(1-8)?

Press "7" to plot. The program will first draw a direction field and then trace the integral curve through the point (0,1) given by the initial condition. Note how the curve's existence and uniqueness is suggested by the field of direction lines. The finished graph is shown below:

MH PLOT3

X0=0

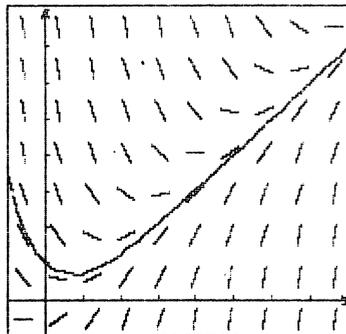
Y0=1

ΔX=1

ΔY=1

Ymin =
.6947

Ymax =
7.0000



X: [-1,8]
Y: [-1,8]

dy/dx=X-Y

When the plotting is done, press any key to return to the menu.

Most options in MH PLOT3 work as in other ARBPLOT programs. Some special considerations are discussed below.

1. DY/DX =. When you select this option, you will be prompted to enter a function. The function should be expressed in terms of X (the independent variable) and Y (the dependent variable) and must make no reference to other variable names.

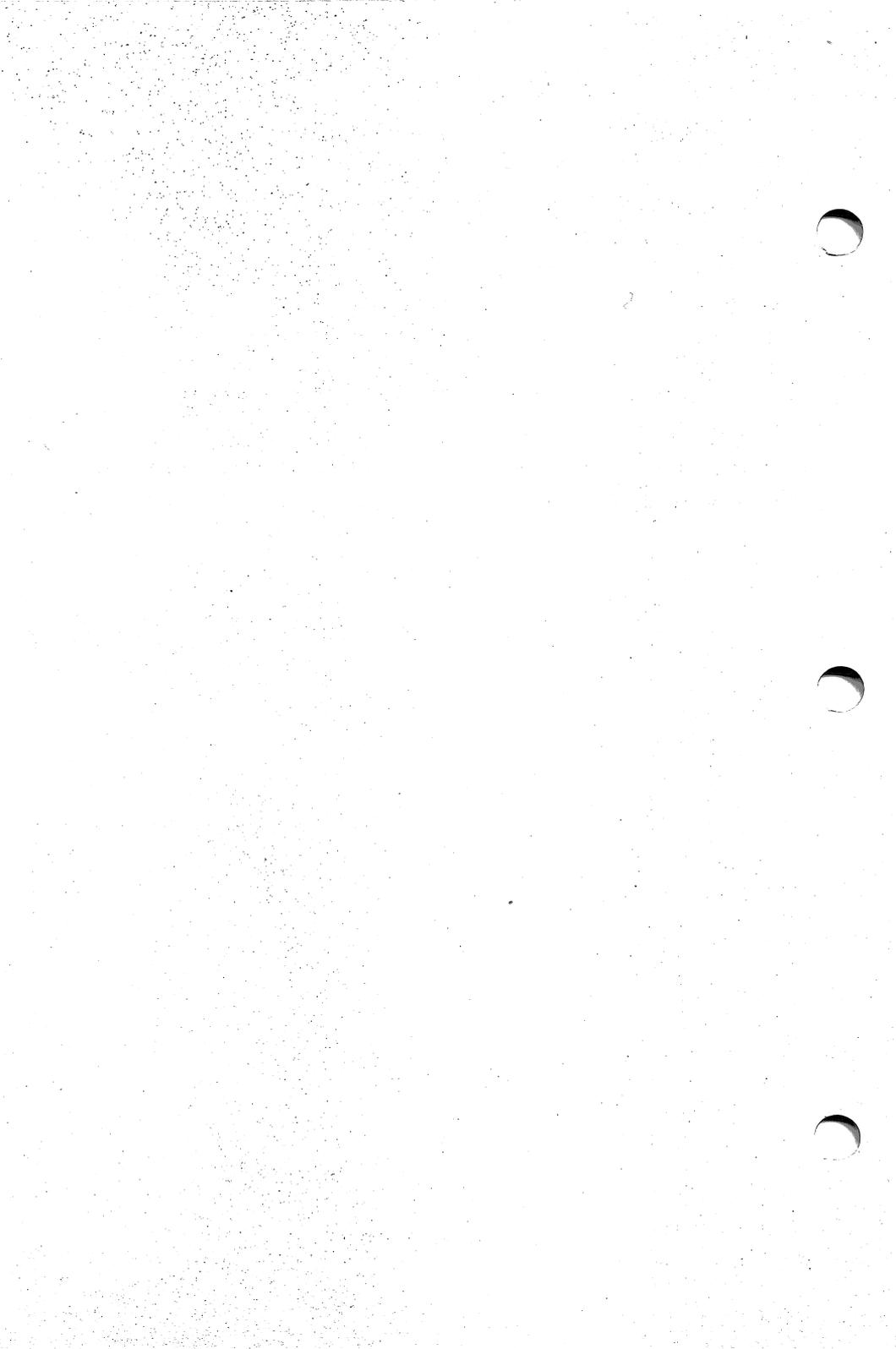
2. INITIAL CONDITION. The X and Y coordinates of some point (within the region specified by option 4 at the time of plotting) are required if an integral curve is to be drawn. These are listed as X0 and Y0 on the final graph.

3. MODE. The choice is from (a) direction field, (b) integral curve, and (c) both. If you select (a), you do not have to specify an initial condition.

4. LIMITS. The scaling can be manual or automatic. If you use manual, you must give both the domain and range; if you use automatic, you enter only the domain. Automatic scaling will be rejected if you select option (a) of MODE. In most cases (if options (b) or (c) are used), automatic scaling will provide at least a first estimate for an appropriate range. This range will be given within brackets on the graph and on the menu (after you return from the graph). If you want to do multiple plots on the same graph, and you use automatic scaling for the first plot, for subsequent plots change the scaling to manual and set the range to that determined by the program in automatic mode. This can be done conveniently by just pressing the return key when prompted for the range (which then defaults to the last range used).

Descriptive Data on the Graph. If an integral curve has been drawn, the initial condition is given as (for example) $X_0=0$, $Y_0=1$, and the maximum and minimum Y-coordinates actually plotted are given as Y_{min} , Y_{max} . The domain X: $[-1,8]$, $[-1,8]$, and the distance between tic-marks is specified by the values of ΔX and ΔY .

Note: The integral curve is plotted in each direction only so long as the function $DY/DX=F(X,Y)$ is defined. Points at which $F(X,Y)$ is not defined are indicated in a direction field by crosses; you can interpret these as vertical tangents in many cases.



4. Tactics: When to Use ARBPLOT

ARBPLOT offers graphic treatment of many topics in two dimensional differential and integral calculus (single independent variable) and related subjects. You can use it dynamically, on line with the Apple II computer, or you can prepare a slide show, transparencies, or class notes. Computer graphics in the mathematics classroom or in individual study can be a powerful aid to learning; but it can also be counterproductive.

When should ARBPLOT be used? Which of the several modes of use is best for a particular situation? There is no single definitive answer. This chapter addresses some of the considerations which can help you decide.

First, do you have the physical resources? If you are to use ARBPLOT in the classroom, be sure the students can see. A single small screen in front of a large class frustrates the students. Large video monitors, television sets, or video projectors are needed, placed where the students can read the text on the screen, as well as follow the curve drawing. Lighting must be such that glare or ambient lighting level does not interfere with viewing. If you cannot provide suitable viewing conditions, use ARBPLOT with a printer and copier to prepare transparencies for an overhead projector. Transparencies are also useful when static graphic results are a part of a larger presentation.

On-line interactive computer graphics can most effectively be used when the topic is inherently dynamic, when the process of curve drawing is intrinsically interesting, or the dynamics of the process being portrayed is an essential part of the process -- such as the first time the students see a Lissajous figure or a cosine rose, or the processes portrayed in the DERIVATIVE FUNCTION and INDEFINITE INTEGRAL programs. On-line interactive graphics are least effective when lengthy calculations and plotting are used to achieve a result which will be discussed only in its final, static form, as in the Fourier Series problem in Chapter 5.

Static results can best be shown by slides or transparencies. If exact registration of a succession of related images is important, a slide show is preferable. If rapid sequencing of a few images is important, then a compressed slide show is preferable.

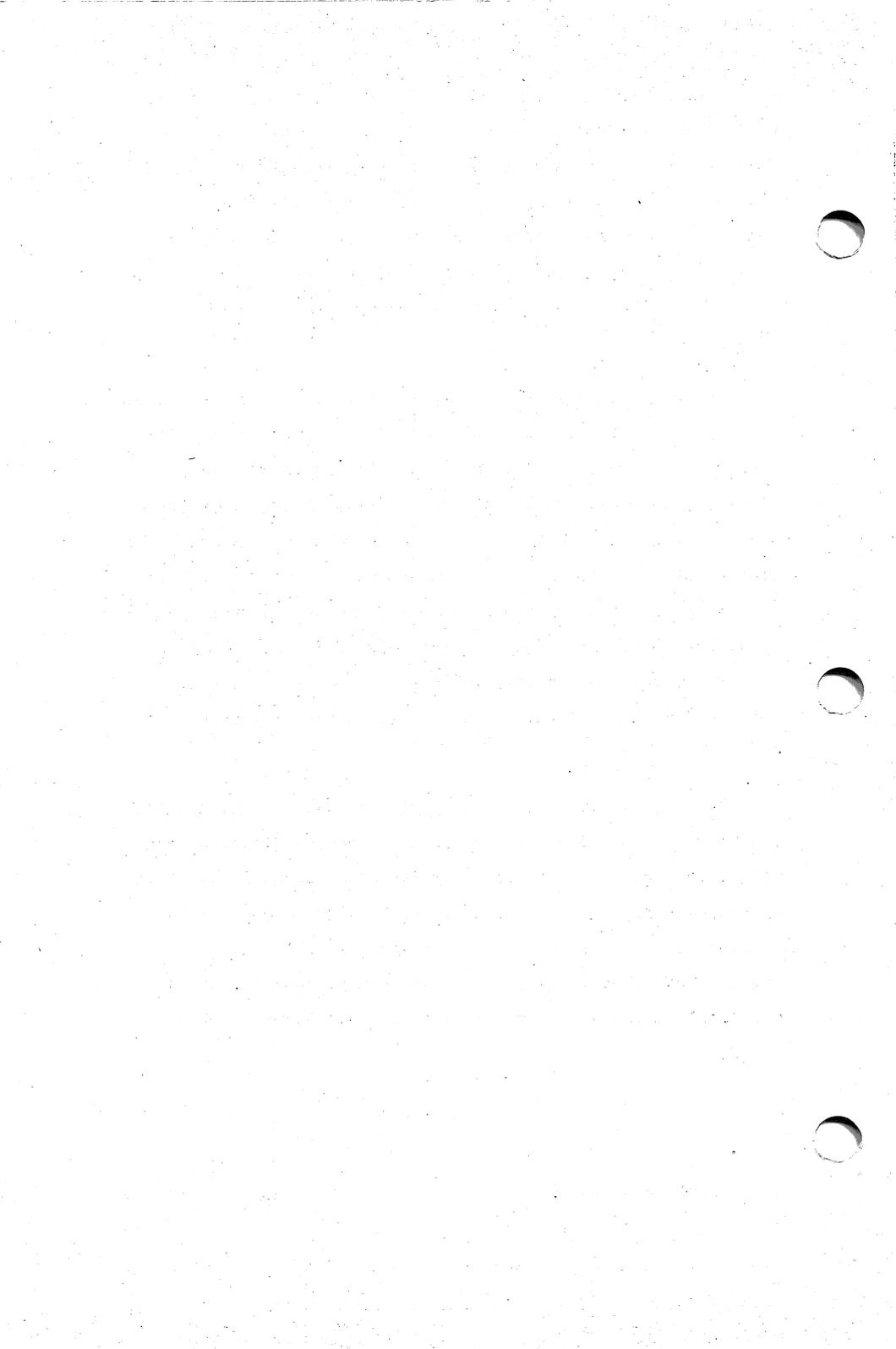
There are several tradeoffs between the two types of slide shows. A regular hi-res image occupies 34 sectors on disk and takes 10 seconds to load. A typical ARBPLOT compressed graph occupies 10-12 sectors; four or five can be combined into one compressed slide show; after 10-15 seconds for loading the show, the slides can be cycled every 0.1 second. Busy ARBPLOT slides may only compress to 30 sectors; compression is not useful. If the images have little blank space or if the minimum discussion time for each slide is more than 10 seconds, then a regular slide show is preferable.

It is important that your use of computer graphics be closely related to your lesson objectives. Careful lesson planning is needed. ARBPLOT computer graphics illustrate

principles, concepts, and processes; they do not teach skills. Therefore, it is essential that graphics presentations be tied closely to skill teaching. For example, ARBROOT shows how Newton's Method works. Its use should be coordinated with algebraic presentation of the process and student practice in using the process with calculators or writing computer programs.

Use of computer graphics should be relevant to the course; there is a temptation to become enamored with graphics. Topics should be chosen for their importance to the course, not primarily for their pictorial value. Many illustrations are given in the next chapter. As discussed above, dynamics is an important consideration in selecting the topic and method of presentation. Also consider the precision the computer can give, its ability to show relationships, and its ability to present solutions that are otherwise too complicated to consider in the limited time available in most calculus classes.

Individual students and even classes can use the ARBPLOT programs in a discovery and enrichment mode, addressing "what if" questions. For example, in polar coordinates $R = \cos kt$ is a rose; $R = A + B \cos t$ is a cardioid. What is $R = A + B \cos kt$; what happens if k is not an integer? In another vein, given a result, what produced it, or how can you produce a desired graphical result? (See the last problem in Chapter 5.)



5. Problems

In Chapter 1 we presented the scope of ARBPLOT. In Chapters 2 and 3 we showed you how to run the programs, including making "slides" or graphic output to a printer. In Chapter 4 we made some suggestions about style of use. Now we turn to specific teaching examples, lessons or exercises you can use in teaching calculus.

Here are thirty examples. They all follow a common pattern. First we pose a problem, then we show the setup for one or more runs of ARBPLOT programs with a sample display. Finally we discuss the displays or process and make suggestions for additional activity by students or instructor. We suggest that you read the discussion before running the program; frequently the discussion parallels the steps you take.

The thirty examples are just that -- examples. They do not pretend to cover the scope of calculus or of computer graphics applications in calculus. We hope these examples will be useful, but more important, that they will give you ideas for developing your own lessons.

There are important topics in the programs in addition to those appearing in the problems. For example, Mean Value Theorem, Typical Element in Integration, Numerical Integration (Trapezoidal Rule, Simpson's Rule, Richardson Extrapolation), and Arc Length are complete programs or portions of programs with instructions given in Chapter 3, even though they do not appear in this chapter. Other subjects which are major parts

of the package, such as Sequences and Series or Manipulation of Quadratic Equations, are touched on only lightly in the problems of this chapter. You may get additional ideas for problems from other examples in Chapter 3 or by experimenting with the programs.

Problem 1. Introducing Function Plotting

The simplest approach to plotting the graph of a function is to plot points, although this alone does not give many properties of the curve. ARBPLOT1 plots points. We shall use this program to plot several curves in order to raise questions about properties of the functions.

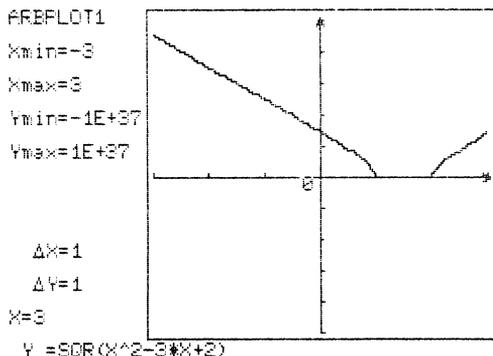
Setup

Disk 1, select #1, PLOT Y(X).

Set Xmin=-3, Xmax=3, DELTA X=.1, scaling=MANUAL, Ymin=-5, Ymax=5. Then PLOT each of the following functions.

- (1) $Y(X)=X^3-3*X$
- (2) $Y(X)=.2*X^4+X^3-.4*X^2-3*X+1$
- (3) $Y(X)=\text{SQR}(X^2-3*X+2)$
- (4) $Y(X)=2*X/(X-1)$
- (5) $Y(X)=\text{SQR}(6+X-X^2)$

Display



Discussion

For each of the functions, what does the curve tell about the domain, range, number and location of extrema, intercepts, symmetries, behavior at infinity? Is the testimony of the curve correct? The intent of this exercise is to motivate students to analyze a function rather than depend completely on plotting points, such as with a computer plotting program.

Activities

Use your own favorite examples.

Problem 2. Introducing Asymptotes

A function has a vertical asymptote at $x=a$ if $\lim_{x \rightarrow a} f(x) = \pm\infty$. It has a horizontal asymptote if $\lim_{x \rightarrow \pm\infty} f(x) = L$. We can illustrate both types by plotting rational functions.

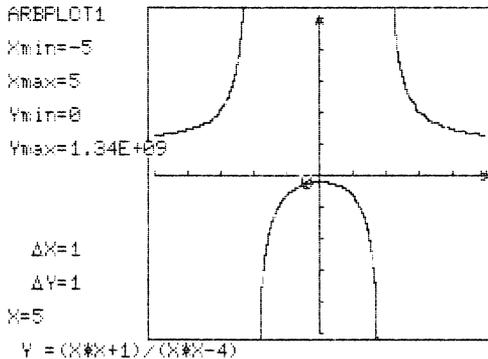
Setup

Disk 1, select #1, PLOT Y(X).

(1) Set $Y(X)=(X*X+1)/(X*X-4)$, $Xmin=-5$, $Xmax=5$, $DELTA X=.1$, $scaling=MANUAL$, $Ymin=-5$, $Ymax=5$. Then PLOT.

(2) Change function as described below and replot.

Display



Discussion

We see the asymptotes at $X=\pm 2$; it seems plausible that $Y=1$ is a horizontal asymptote at both $+\infty$ and $-\infty$. If we extend the domain, say to $-10 \leq X \leq 10$, we make the horizontal asymptotes more plausible. The plots should be combined with algebraic

analysis to prove that we have actually seen the asymptotes.

Activities

Experiment with changes in the function and see the effects on the graph. For example, with the same limits, try $Y(X)=X*(X+2)/(X*X-4)$, $X*X/(X*X-4)$, $X/(X*X-4)$, $X^3/(X*X-4)$, or others of your choice.

What are the effects of the numerator being of the same, lower, or higher degree than the denominator?

Try some transcendental functions, e.g., $EXP(X)$, $LOG(X)$, $TAN(X)$, $1/COS(X)$ (= sec x). How do you prove the apparent asymptotes really exist? How do you locate them exactly?

Note: $X*X$ is more accurate and faster than X^2 .

Problem 3. Introducing Inverses

$G(X)$ is the inverse of $F(X)$ if $F(G(X))=X$. ARBPLOT4 plots an inverse of $Y(X)$ simply by interchanging X and Y . We shall plot the inverse in this fashion (called mechanical inverse), then compare it with an inverse function.

Setup

Disk 1, select #4, PLOT $Y1(X), Y2(X)$, SIMULTANEOUSLY.

(1a) Set $Y1(X)=\text{EXP}(X)$, $Y2(X)=\text{INVERSE}$, $X_{\min}=-2$, $X_{\max}=6$, $\text{DELTA } X=.1$, $\text{scaling}=\text{MANUAL}$, $Y_{\min}=-2$, $Y_{\max}=6$, $\text{screen}=\text{CLEAR}$, $Y1$ SOLID, $Y2$ DOTTED, NO VERTICALS. Press "7" to PLOT.

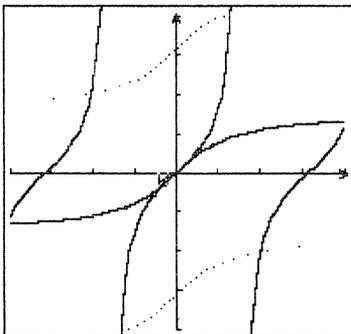
(1b) Change $Y2(X)=\text{LOG}(X)$, $\text{screen}=\text{KEEP}$, $Y2$ SOLID. Press "7" to PLOT.

(2a) Set $Y1(X)=\text{TAN}(X)$, $Y2(X)=\text{INVERSE}$, $X_{\min}=-4$, $X_{\max}=4$, $\text{DELTA } X=.1$, $\text{scaling}=\text{MANUAL}$, $Y_{\min}=-4$, $Y_{\max}=4$, $\text{screen}=\text{CLEAR}$, $Y1$ SOLID, $Y2$ DOTTED, NO VERTICALS. Press "7" to PLOT.

(2b) Change $Y2(X)=\text{ATN}(X)$, $\text{screen}=\text{KEEP}$, $Y2$ SOLID. Press "7" to PLOT.

Display

```
ARB PLOT4
Xmin=-4
Xmax=4
Y1min=-34.2
Y1max=34.2
Y2min=-1.33
Y2max=1.33
  ΔX=1
  ΔY=1
X=4
Y1=TAN(X)
Y2=ATN(X)
```



Discussion

(1a) gives the mechanical inverse, reflection in the 45 degree line; it is a function (single valued). What function? Try $Y2(X)=\text{LOG}(X)$ in (1b) and you will see it is the desired function, as shown by its plot overlaying that of the mechanical inverse.

(2a) gives the mechanical inverse of $\text{TAN}(X)$, but it is not a function (multiple valued). Try $Y2(X)=\text{ATN}(X)$ in (2b) and fill in only one curve, the principal value.

For algebraic examples, we could use in (1):

$$Y1(X)=X^3, Y2(X)=\text{SGN}(X)*(\text{ABS}(X))^{.33333}$$

and in (2):

$$Y1(X)=X^2, Y2(X)=\text{SQR}(X), \text{ with } -1 \leq X \leq 1.$$

Activities

(1) Try $Y1(X)=X^3+1$. Does it seem to have an inverse function? How can you prove it does (or doesn't)? Can you find the inverse if it exists, or find the inverse on a suitable restricted domain?

(2) Same questions for:

$$Y1(X)=X^3+X+1$$

$$Y1(X)=X^3-1$$

$$Y1(X)=X^3-X+1$$

Problem 4. Introducing Limits

What does the epsilon-delta definition of a limit mean? How does it work? Let's demonstrate the process, first with a continuous function, then with a function with a "hole" in it, and finally with a function with a finite discontinuity (jump).

Setup

Disk 1, select #5, LIMITS & DERIVATIVES.

(1) Change only #3, the problem, to LIMIT. Set the LIMIT to 1.5, press "7" to PLOT. Press "C" to CONTINUE until the message "NOT THE LIMIT" appears. EXIT by pressing "E", press "3" again, accept LIMIT, but press "Y" to set the limit to Y(1). Press "7" to PLOT. Continue through at least one ZOOM ("Z") cycle.

(2) Change the function to

$$Y(X)=(X\leq.999 \text{ OR } X\geq 1.001)*X*X+(X>.999 \text{ AND } X<1.001)*1.2.$$

Set Xmin=.85, Xmax=1.15, LIMIT=1. You can take several steps before the limitations of the way BASIC defines an isolated point make the test fail.

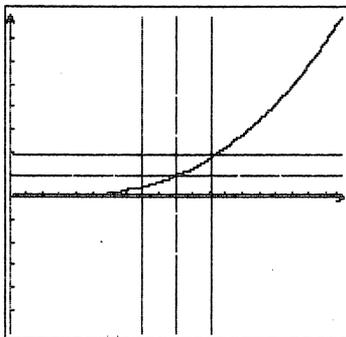
(3) Change the function to $Y(X)=X*\text{INT}(X+1)$ and the interval to $0.5\leq X\leq 1.5$. Try the LIMIT as Y(1), then 1, then 1.5. Continue until the message "NOT THE LIMIT" appears.

Display

```
ARBLIM
Limit
Xmin=0
Xmax=2
Ymin=0
Ymax=8

ΔX=.1
ΔY=1

Y = X^3
XL=1
EPS=.875
```



LIMIT=1

DELTA=.208

Discussion

We can use computer graphics to illustrate two features of the epsilon-delta definition of the limit. First, the program shows that the definition will not find a limit; it strictly tests a guess. Second, the program demonstrates the "squeezing" process. Discuss in detail the first few steps of the process showing that one, but usually not both, ends of the epsilon interval will determine delta. Use all of the examples to show that there are many ways for the test to fail, but only one for success. ARBLIM cannot illustrate all types of functions, only ones which are monotonic near the limiting value of x .

Activities

Try other functions and other intervals. Your function must be monotonic except that you can show a few steps toward convergence of a function with an isolated point.

Problem 5. Introducing Derivatives

The derivative is initially defined as the limit of a geometric ratio. We shall use computer graphics first to show this limit at a point, then to show tangents to a curve across an interval and to plot their slopes, and finally to get the derivative function and plot it.

Setup

(1) Disk 1, select #5, LIMITS & DERIVATIVES.

Run the built-in example. PLOT, then repeatedly press "<" to move the secant until DELTA-X=0. Press "L" to move the crossing point to the left, then repeatedly press ">" until DELTA-X=0.

Set $Y(X)=X^3-3*X$, Xmin=-1, Xmax=2, repeat.

Set $Y(X)=\text{SQR}(\text{ABS}(X))$, Xmin=-1, Xmax=1, repeat.

(2) Disk 1, select #6, DERIVATIVE FUNCTION WITH EXTREMA.

Set $Y(X)=X^3$, Xmin=-2, Xmax=2, scaling=AUTOMATIC. Then PLOT.

Set $Y(X)=X^3-3*X$. PLOT.

Set $Y(X)=\text{SQR}(\text{ABS}(X))$. PLOT.

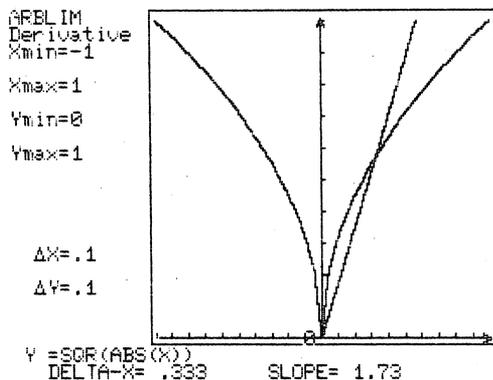
(3) Disk 1, select #4, PLOT Y1(X),Y2(X), SIMULTANEOUSLY.

Set $Y1(X)=X^3$, $Y2(X)=3*X^2$, Xmin=-2, Xmax=2, scaling=AUTOMATIC, AUTOMATIC VERTICALS on Y2. Then PLOT.

Set $Y1(X)=X^3-3*X$, $Y2(X)=3*X^2-3$. PLOT.

Set $Y1(X)=\text{SQR}(\text{ABS}(X))$, $Y2(X)=\text{SGN}(X)/(2*\text{SQR}(\text{ABS}(X)))$, scaling=MANUAL, Ymin=-3, Ymax=3. PLOT.

Display



Discussion

(1) First, we demonstrate the process of finding the slope of the tangent as the limit of slopes of secants.

(2) Next, we plot the function over an interval and plot the slope of the tangent as a function of X over the same interval. Can you anticipate the derivative function from the shape of the "slope of tangent" curve?

(3) Finally, we differentiate the function and plot the function and its derivative simultaneously over the interval.

We have demonstrated the derivative at a point, a geometrically obtained derivative function, and a derivative function obtained by differentiation. We have seen that a continuous function need not have a continuous derivative. As a byproduct, we have seen extrema of the function corresponding to zeroes and discontinuities of the derivative.

Activities

Repeat the exercise with some transcendental functions,
e.g., $\text{TAN}(X)$, $\text{SIN}(X)$, $\text{EXP}(X)$, $\text{LOG}(X)$.

Problem 6. Geometric Derivatives

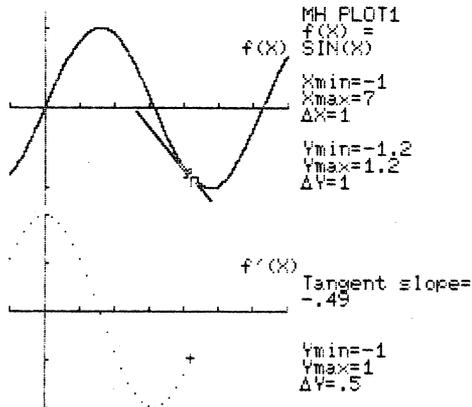
Show graphically the plausibility of $d(\sin x)/dx = \cos x$ by "freezing" tangent lines at various points on the graph of $Y(X) = \text{SIN}(X)$.

Setup

Disk 1, select #6, DERIVATIVE FUNCTION WITH EXTREMA.

Run the built-in example. While the derivative is being graphed, freeze the action occasionally.

Display



Discussion

Do this demonstration before deriving the formula for the derivative of $\sin x$.

$\sin x$ is increasing on the intervals $[-\pi/2, \pi/2]$, $[3\pi/2, 5\pi/2]$, etc. and decreasing elsewhere. Extrema occur at $\pi/2, 3\pi/2$, etc. These are reflected on the graph of the

derivative as intervals on which the derivative is positive or negative, and as points at which the derivative is zero (and changes sign). Verify that these intervals and sign are consistent with $d(\sin x)/dx = \cos x$.

Activities

This approach can be taken before deriving the formula for differentiating any new function, e.g., quadratic and cubic polynomials, $TAN(X)$, $EXP(X)$, $LOG(X)$.

Problem 7. Extrema of Cubics

In this problem, we shall use computer graphics to characterize the extrema of cubics.

Setup

(1) Disk 1, select #6, DERIVATIVE FUNCTION WITH EXTREMA.

Set $X_{min}=-2$, $X_{max}=2$, scaling=AUTOMATIC.

Set $F(X)=X^3-3*X$. PLOT.

Set $F(X)=X^3+3*X$. PLOT.

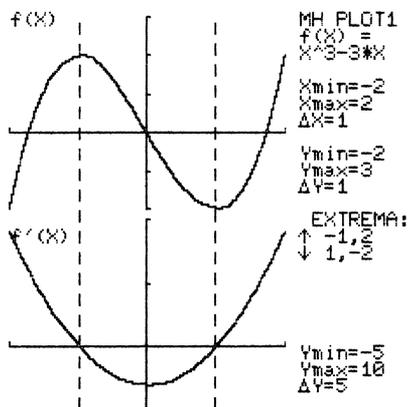
Set $F(X)=X^3$. PLOT.

(2) Disk 1, select #4, PLOT $Y1(X), Y2(X)$, SIMULTANEOUSLY.

Set $X_{min}=-2$, $X_{max}=2$, $\Delta X=.1$, scaling=AUTOMATIC,
AUTOMATIC VERTICALS on Y2.

Set $Y1(X)$ = each of the functions in (1), with $Y2(X)$ = the derivative function. Then PLOT.

Display



Discussion

If $f(x)$ is a polynomial of degree n , then $f'(x)$ is a polynomial of degree $n-1$ and has at most $n-1$ zeroes. This means that $f(x)$ has at most $n-1$ local extrema. Furthermore,

(1) if $f(x)$ is of odd degree, then $\#max = \#min$, and

(2) if $f(x)$ is of even degree, then $\#max = \#min \pm 1$.

In the case of a cubic, this means that either

(1) there are no extrema, or

(2) there are exactly one maximum and one minimum.

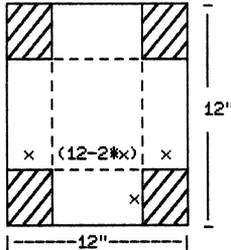
For any polynomial, $f'(x)=0$ is a necessary but not sufficient condition for an extremum.

Activities

A quartic will have at most three extrema. Construct and graph quartics with exactly one and exactly three extrema. Is it possible to construct a quartic with an even number of extrema? Why?

Problem 8. Applied Maxima/Minima

An open box is to be constructed from a 12"x12" sheet of cardboard by cutting away corners and folding.



What are the dimensions of the box resulting in the greatest possible volume?

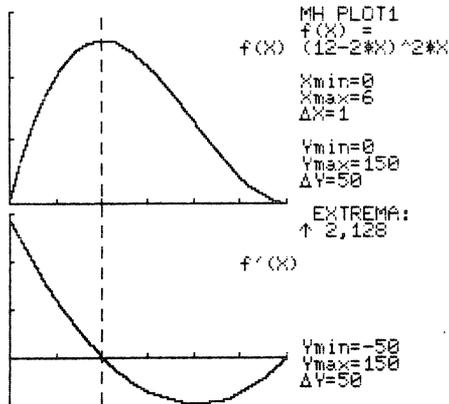
If we let X =length of square to be cut away, then we must maximize $(12-2*X)^2*X$ on the domain $[0,6]$.

Setup

Disk 1, select #6, DERIVATIVE FUNCTION WITH EXTREMA.

Set $F(X)=(12-2*X)^2*X$, $Xmin=0$, $Xmax=6$. Then PLOT.

Display



Discussion

The graph shows a local (and absolute) maximum at $X=2$. Thus the optimal dimensions are $8 \times 8 \times 2$. Note that $X=2$ is the only zero of the derivative between 0 and 6. (There is another zero at $X=6$ corresponding to zero volume, a minimum.) In many max/min problems, the following line of reasoning can be used:

- (1) A continuous function must have a maximum and minimum on a closed and bounded domain.
- (2) The derivative must be zero or undefined at an extremum.
- (3) There is only one such candidate for the maximum or minimum sought.
- (4) Therefore, the candidate is the solution.

This can often reduce the problem of finding the optimal value of a function to the problem of finding a unique zero of the derivative on a given domain (as opposed to graphing or checking the sign of the derivative).

Activities

Use the same technique to find the dimensions of a cylindrical container (closed bottom, open top) using minimum material (assume no waste) to hold one liter.

Problem 9. Introducing Roots

For what value of x does $x = \cos x$? We shall use this question to introduce several approaches to finding roots of an equation.

Setup

(1a) Disk 1, select #4, PLOT Y1(X),Y2(X), SIMULTANEOUSLY.

Set $Y1(X)=X$, $Y2(X)=\cos(X)$, $Xmin=0$, $Xmax=1.57$, $\Delta X=.1$, $scaling=AUTOMATIC$, both curves SOLID. Then PLOT.

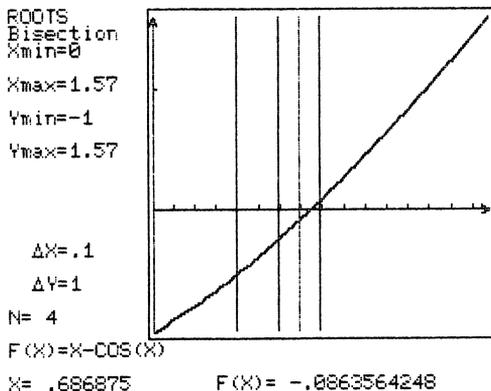
(1b) Disk 1, select #1, PLOT Y(X).

Set $Y(X)=X-\cos(X)$, $Xmin=0$, $Xmax=1.57$ $\Delta X=.1$, $scaling=AUTOMATIC$. PLOT.

(2) Disk 1, select #7, ROOTS OF $F(X)=0$.

Set $F(X)=X-\cos(X)$, $Xmin=0$, $Xmax=1.57$. In turn, use BISECTION, REGULA FALSI, NEWTON.

Display



Discussion

To see where two functions have the same value, we might plot their graphs and see where they intersect (1a). We could just as well plot their difference and see where it crosses the X-axis (1b). In either case, X is about .7. To improve the estimate, we might replot for $.7 \leq X \leq .8$ with $\text{DELTA } X = .01$, and see that X is about .74. Methods for root finding are orderly ways to improve the estimate.

Bisection takes the new estimate halfway between previous ones. Regula Falsi uses linear interpolation to find the next estimate. In each case, we start with the function having opposite signs at the previous estimates and discard one of the old estimates after obtaining the new one. Newton's Method only needs one previous estimate but must know there is a root to be found.

We can start as suggested in (2) with the same limits as we used when beginning (1), or we can use ARBROOT just for improving the estimates found in (1). We soon find $X = .739085$ as the root to six significant figures.

Activities

Derive the formulas and procedures for the three methods. Compare the number of iterations required for convergence for the three methods. When would you choose each method?

Study a book on Theory of Equations to find methods for determining the number of real roots of an equation and isolating them.

Be sure to do sample calculations by hand or have the students write their own root-finding programs to relate the demonstration to developing root-finding skills.

Problem 10. Newton's Method

We shall use the examples of Section 3.2.3 to explore the method, power, and limitations of Newton's Method for finding roots of an equation.

Setup

Disk 1, select #7, ROOTS of $F(X)=0$.

(1) $F(X)=X^2-5$; run built-in example.

(2) $F(X)=\cos(X)$; $F'(X)=-\sin(X)$, $X_{\min}=0$, $X_{\max}=4$, then

FIND ROOT.

(3) $F(X)=X^3-X+1$; $F'(X)=3X^2-1$, $X_{\min}=-2$, $X_{\max}=2$, then

FIND ROOT.

Display

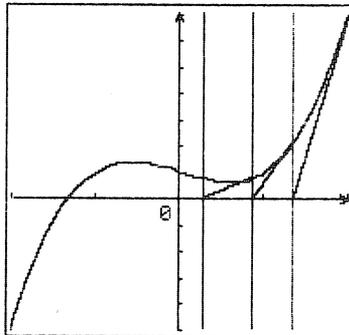
```
ROOTS
Newton
Xmin=-2
Xmax=2
Ymin=-5
Ymax=7
```

```
ΔX=1
ΔY=1
N= 3
```

```
F(X)=X^3-X+1
```

```
X= .296894917
```

```
F(X)= .729864376
```



Discussion

Use (1) to show successful use of Newton's Method. Use (2) to show how Newton's Method can lead you away from a root.

Use (3) to show how Newton's Method can try to find a root which doesn't exist.

Activities

Derive Newton's Method. By hand, start at $X=4$ in example (2) and see where it leads you. By hand, carry Newton's Method from the right in example (3) beyond the point at which it switches to the left end of the interval. Observe oscillation.

ARBROOT uses two rules to guarantee convergence of Newton's Method (switch to left, bisection). Can you think of others?

Problem 11. Transformation of Coordinates

What is the effect on the coefficients of an equation of moving the coordinate axes parallel to the original axes? What is the effect of rotating axes? What transformation will reduce the equation to its simplest form?

Setup

(1) Disk 2, select #1, CONIC SECTIONS.

Change PROBLEM to TRANSLATE AXES, then PLOT.

Move X axis down, then up, then down repeatedly until the Y term vanishes.

Move Y axis to the right, then left, then right repeatedly until the X term vanishes.

(2) Disk 2, select #2, ROTATION OF AXES; ELLIPSE.

Run the built-in example.

Display

CONIC
Ellipse

Xmin=-2

Xmax=4

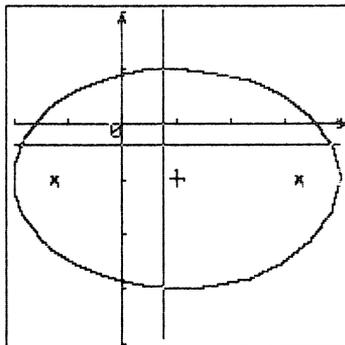
Ymin=-2.99

Ymax=.995

$\Delta X=1$

$\Delta Y=1$

I = 64



$4X^2 + 9Y^2 - 2X + 11.3Y - 32.2 = 0$

$(.75, -.375)$

Note: See also figure in Section 3.4.2.

Discussion

In each case, first we demonstrate the process, then we seek the particular transformation which will simplify the equation. Show the geometry and derive the equations for the transformations; use the tutorial in the rotation program.

Activities

Set up equations in STANDARD QUADRATIC form for a circle, an ellipse, a hyperbola, and a parabola with center (vertex for parabola) (H,K) not at the origin. Convert to GENERAL QUADRATIC form; translate axes to transform equation to simplest form. Show that coordinates of the new origin relative to the original are (H,K) .

Problem 12. Sum and Difference Definitions of Conic Sections

One way to define an ellipse is as the locus of a point moving so that the sum of its distances from two fixed points is constant. Similarly, a hyperbola can be defined as the locus of a point moving so that the difference of its distances from two fixed points is constant. Let's see the process and the curves and equations produced.

Setup

Disk 2, select #1, CONIC SECTIONS.

(1) Set axes=DON'T SHOW. Set problem=SUM OR DIFFERENCE, CONSTRUCT NEW CURVE. Choose SUM, DISTANCE=4, SUM=12. Then PLOT.

Set screen=KEEP, accept the limits. Reselect problem=SUM OR DIFFERENCE, CONSTRUCT NEW CURVE. Choose SUM, keep DISTANCE=4. Successively set SUM=8, 5, 4.4. PLOT each new ellipse.

(2) Set screen=CLEAR. Reselect problem=SUM OR DIFFERENCE, CONSTRUCT NEW CURVE. Choose DIFFERENCE, keep DISTANCE=4. Set DIFFERENCE=3.6. Then PLOT.

Set screen=KEEP, accept the limits. Reselect the problem=SUM OR DIFFERENCE, CONSTRUCT NEW CURVE. Choose DIFFERENCE, keep DISTANCE=4. Successively set DIFFERENCE=3, 2, 1. PLOT each new hyperbola.

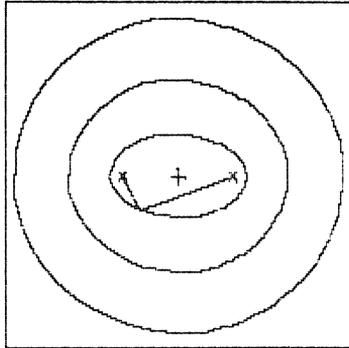
Display

CONIC
Ellipse

Xmin=-2.5
Xmax=2.5
Ymin=-1.24
Ymax=1.5

I = 64

$$\left(\frac{X}{2.5}\right)^2 + \left(\frac{Y}{1.5}\right)^2 = 1$$



Discussion

Conic sections with constant distance between the fixed points are called confocal. For confocal ellipses and hyperbolas, notice how the shape and size change as the sum/difference changes.

When using SUM OR DIFFERENCE while plotting a family of ellipses or hyperbolas, always work from the outside in to minimize the erasure of earlier curves. If the desired result is just the curves, not the animation, use problem: DRAW; there will be no erasures. You can use SUM OR DIFFERENCE to generate the equation, then switch to DRAW to plot.

Activities

For ellipses, what is the result if SUM=DISTANCE, if SUM is much larger than DISTANCE?

A dome with elliptic cross section is called a "whispering dome" because a sound produced at one focus can be heard clearly at the other focus. Explain this phenomenon.

Confocal ellipses and hyperbolas are mutually perpendicular. Why? (Compare problem 29.)

Note: This exercise might well be done as a dynamic presentation of one ellipse and one hyperbola, then as slide shows of confocal ellipses, confocal hyperbolas, and confocal ellipses and hyperbolas, perhaps using MH GRAPH EDITOR to label the distances.

Problem 13. Eccentricity Definition of Conic Sections

If a point moves so that the ratio of its distances from a fixed point and a fixed line is constant, it sweeps out a conic section. Let's see the process and the curves and equations generated. We shall work in polar coordinates, but the equations may also be examined in standard quadratic form.

Setup

Disk 2, select #1, CONIC SECTIONS.

Set problem=ECCENTRICITY, scaling=MANUAL, Xmin=-10, Xmax=10, Ymin=-10, Ymax=10, axes=DON'T SHOW.

At #1, choose POLAR, reject current equation, set ECCENTRICITY=.5, DISTANCE=2, DIRECTION:L. Then PLOT.

Set screen=KEEP, change equation successively to ECCENTRICITY=.7, .9, 1, 1.5, 2, 4. Keep DISTANCE=2, DIRECTION:L. PLOT after each change.

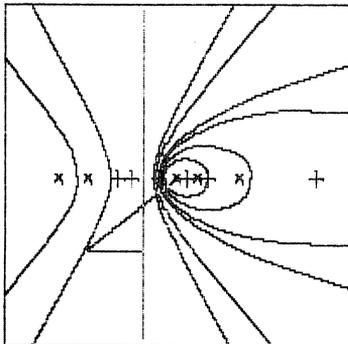
Display

CONIC
Hyperbola

Xmin=-5.61
Xmax=-4
Ymin=-4.55
Ymax=8

I = 64

$R(T) = 2*2/(1 - 2*\text{COS}(T))$



Discussion

The directrix and one focus are fixed. As the eccentricity increases, the other focus moves to the right toward $+\infty$, disappears altogether for the parabola, then reappears from the left for the hyperbolas.

Activities

Prove that a parabolic reflector concentrates parallel axial rays at a single point.

Problem 14. Area

What is area? We know and can calculate area for figures composed of rectangles and triangles, but what of irregular, curved figures? As a preliminary to defining the definite integral, let's examine two approaches to defining the area of a figure bounded by curved lines.

Setup

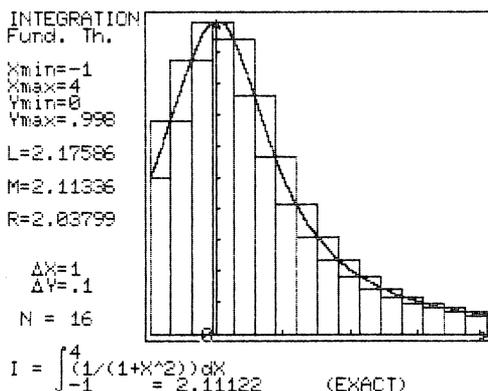
(1) Disk 2, select #5, MONTE CARLO INTEGRATION.

Run the built-in example. CONTINUE once with DOUBLE (via #3 NUMBER OF DARTS).

(2) Disk 2, select #3, DEFINITE INTEGRAL.

Set method=RECTANGLES, then INTEGRATE. Change number of subdivisions successively to 8, 16, 32, and INTEGRATE each time.

Display



Discussion

One unconventional approach to defining area is as a ratio (see Section 3.5.3). The first part of this exercise is intended to get students to think about area. Second, we approximate area as the sum of rectangular strips, but which strips? The left, middle, and right rectangles let us consider the variation due to choice of the point at which we get the height of a rectangle. In both parts of the exercise, we examine the effect of increasing the sample size, the number of "darts" or rectangles.

One dart or one rectangle requires one function evaluation. Which is more effective? We need a standard. At the bottom of the screen we are given the "exact" value. At this stage we need not know the source of that value, we just use it to show that computational effort is generally more effective with rectangles than darts, and that more calculations bring us closer to the exact value.

Activities

Try a function/interval combination with the function symmetric about the midpoint of the interval and with the function anti-symmetric.

Problem 15. The Definite Integral

The definite integral is defined as

$$\int_a^b f(x) dx = \lim_{\|P\| \rightarrow 0} \sum_{i=1}^n f(w_i) \Delta x_i$$

where $P = \{\Delta x_i\}$ and $\|P\|$ is its largest element. We shall illustrate the processes used in developing this definition.

Setup

Disk 2, select #3, DEFINITE INTEGRAL.

Set method=RECTANGLES, Xmin=0, Xmax=3. INTEGRATE first with 4 subdivisions, then 8, 16, 32.

Change function to Y1(X)=X*X+1, Y2(X)=0. Repeat the four integrations.

Display

INTEGRATION
Fund. Th.

Xmin=0
Xmax=3
Ymin=0
Ymax=10

L=10.3828

M=11.9648

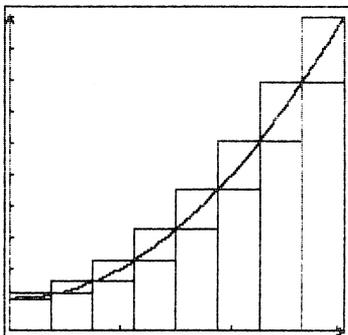
R=13.7578

$\Delta X=1$
 $\Delta Y=1$

N = 8

$I = \int_0^3 (X^2+1) dx$

NO ANTIDERIVATIVE GIVEN.



Discussion

The definition of the definite integral permits using any sequence of partitions, so long as the norm (equal to the largest element) approaches zero. Here we are restricted to having all subdivisions of equal width but we can illustrate other aspects of the definition and of the proof that the limit exists for continuous functions.

First, we take a decreasing function. The left rectangles are upper bounds; the right are lower. We see them coming together with the middle rectangles squeezed in between.

Next, we take an increasing function; the role of left and right is reversed, but the process is the same. If you wish, you can insert an antiderivative to show where the sums are heading.

Activities

Try other functions and differences of functions, for example:

$$Y1(X)=X^3, Y2(X)=X, \text{ for } -1 \leq X \leq 0.$$

$$Y1(X)=4*X-X*X, Y2(X)=X, \text{ for } 0 \leq X \leq 4.$$

$$Y1(X)=X+\text{SIN}(X), Y2(X)=0, \text{ for } 0 \leq X \leq 6.$$

$$Y1(X)=\text{SQR}(X)*\text{INT}(X), Y2(X)=0, \text{ for } 0 \leq X \leq 4.$$

(Note that right-hand value is much larger since $Y1(4)=8$, even though it doesn't show on the screen.)

Problem 16. Extrema in Indefinite Integrals

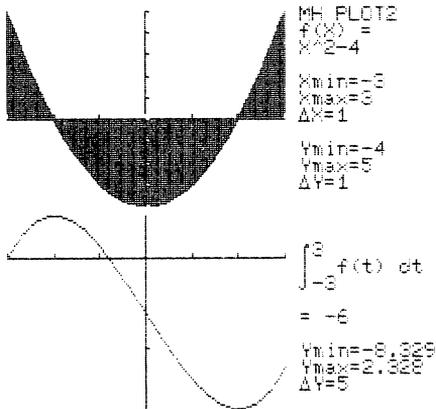
What behavior in $F(X)$ results in extrema or inflection points in the indefinite integral?

Setup

Disk 2, select #4, INDEFINITE INTEGRAL.

Set $F(X)=X*X-4$, $Xmin=-3$, $Xmax=3$. Then PLOT. Interrupt plotting from time to time to discuss progress.

Display



Discussion

Changes of sign of $F(X)$ correspond to extrema of its indefinite integral, and extrema of $F(X)$ correspond to inflection points of the integral. Why? Compare with Problem #7.

Activities

Try other functions, e.g., $F(X)=(X^2-1)/(X^2+1)$, $\text{SIN}(X)$,
 $\text{EXP}(-X)*\text{SIN}(X)$.

Problem 17. Probability Density Functions

We shall show the relationship between a probability density function (pdf) and the associated cumulative distribution function (cdf).

A pdf is a non-negative function whose (improper) integral from negative infinity to positive infinity is one. It is a density function because it is the area beneath the curve rather than a point on the curve itself that gives the probability of an event. In particular, if $f(x)$ is a pdf for a given quantity (such as a grade), then the integral of $f(x)$ from a to b gives the probability that a randomly sampled grade falls between a and b . The requirement that the integral over R be 1 amounts to insisting that the probability of a given grade falling between negative and positive infinity be 1.

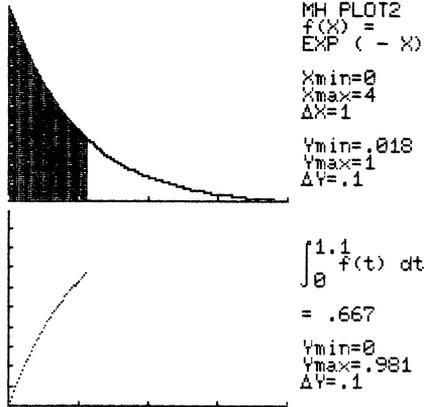
The cumulative distribution function associated with $f(x)$ is the function $g(x)$ whose value at x is the integral of $f(t)$ with respect to t from negative infinity to x . The value $g(x)$ gives the probability that a randomly selected grade (or datum) falls below x . Clearly, $g(x)$ must be a non-negative, increasing function tending to $y=1$ as x is increased.

Setup

Disk 2, select #4, INDEFINITE INTEGRAL.

Run the built-in example.

Display



Discussion

The built-in example of MH PLOT2 can be used to demonstrate a pdf and cdf pair. The function $F(X)$ defined to be $\text{EXP}(-X)$ if $X \geq 0$ and 0 if $X < 0$ is a legitimate pdf, called an exponential distribution. Because of its definition for negative X , its integral from negative infinity to b ($b > 0$) equals the integral of $\text{EXP}(-X)$ from 0 to b . Hence, a left endpoint of zero will be appropriate for the domain of our graph. A right endpoint of 4 will capture most of the interesting activity of both functions.

What kind of physical situation might be represented by this particular pdf? If X is the amount of time a person waits in a line to get served, then the pdf reflects our expectation that

- (1) there is zero probability of being served in negative time, and
- (2) one is more likely to be served in the first 10 minutes

than in the second 10 minutes, more likely to be served in the second 10 than the third, etc.

Activities

For this pdf, what is the probability of being served within .5, 1, 2, 3 minutes? What is the probability of being served during the first minute, the second, the third? Run the example, interrupting to get the value. Also, find the antiderivative and evaluate the appropriate integrals.

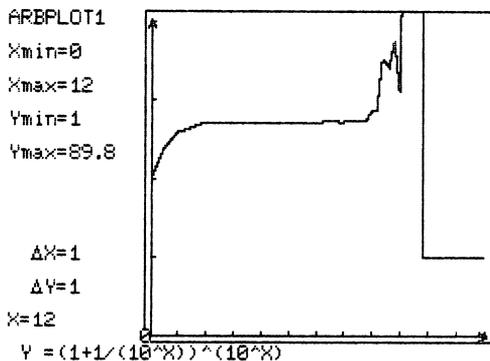
Problem 18. Limit Definition of e

The base for the natural logarithm system can be defined as $e = \lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n$. How is this limit approached? How well can this limit be used to approximate e?

Setup

- (1) Disk 1, select #1, PLOT Y(X).
- (1) Set $Y(X) = (1 + 1/X)^X$, $X_{min} = 1$, $X_{max} = 100$, $\Delta X = 2$, $scaling = MANUAL$, $Y_{min} = 0$, $Y_{max} = 4$. Then PLOT.
- (2) Set $Y(X) = (1 + 1/(10^X))^{(10^X)}$, $X_{min} = 0$, $X_{max} = 12$, $\Delta X = .2$. PLOT.

Display



Discussion

With the first plot, we see Y increasing first rapidly, then slowly, reaching approximately 2.7 when X=100. To move to very large X, replace X with 10^X , then plot again. Peculiar

things happen as X approaches 8 (exponent 10^8); then suddenly at X near 10 (exponent 10^{10}) the function becomes identically one. The cause is the limitations of finite arithmetic, first accumulated roundoff error, then $1/X=0$ if X is large enough. With a computer we cannot get arbitrarily close to a limit.

Activities

(1) Have members of the class use their calculators to calculate $(1+1/X)^X$ for $X=10^7, 10^8, 10^9, 10^{10}$, and 10^{11} . Are there differences from calculator to calculator in the results? What does this say about the way the various calculators internally do arithmetic?

(2) If you have access to a computer which does double precision arithmetic, repeat the calculation on that computer in single and double precision arithmetic. How large must X be for roundoff error to show? For the expression to collapse?

Problem 19. Taylor Polynomials

We shall illustrate successive Taylor (MacLaurin) polynomial approximations to several functions to show the convergence process and its limitations.

Setup

Disk 1, select #4, PLOT Y1(X),Y2(X), SIMULTANEOUSLY.

(1a) Use the built-in example, but set $Y2(X)=X$. Then PLOT.

(1b) Set screen=KEEP. Successively add terms to $Y2(X)$ and PLOT each new polynomial until you reach

$$Y2(X)=X-X^3/6+X^5/120-X^7/5040+X^9/362880.$$

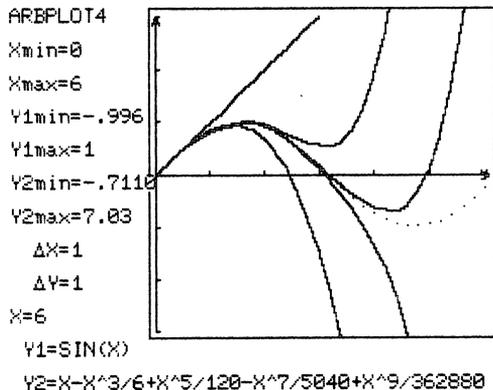
(2a) Set $Y1(X)=1/(1-X)$, $Y2(X)=1+X$, $Xmin=-1.5$, $Xmax=1.5$, $\Delta X=.05$, $scaling=MANUAL$, $Ymin=-5$, $Ymax=5$, $screen=CLEAR$.

Then PLOT.

(2b) Set screen=KEEP. Successively add terms to $Y2(X)$ and PLOT each new polynomial until you reach

$$Y2(X)=1+X+X^2+X^3+X^4+X^5.$$

Display



Discussion

(1) shows the way each successive polynomial extends the interval of a "reasonably good" fit but that eventually a polynomial must depart from a bounded function. Why?

(2) shows what happens when we try to fit past a discontinuity.

Activities

Try $Y1(X)=EXP(X)$, $-2 \leq X \leq 2$, $-1 \leq Y \leq 8$ with $Y2(X)$ the first few terms of the MacLaurin series. PLOT as in (1). The approximation is always less than the function for $X > 0$ but alternates less or greater for $X < 0$. Why?

Try $Y1(X)=TAN(X)$, $-2 \leq X \leq 2$, $-5 \leq Y \leq 5$, $Y2(X)=X+X^3/3+2*X^5/15$ or more terms if you have the patience to calculate them. As in (2) we cross a discontinuity. We cannot get a general formula for the nth term of the series. What would you guess would be the radius of convergence? Why would the MacLaurin polynomial not be a good function to use to approximate $TAN(X)$? What would be good?

At the main ARBPLOT menu, select #1, PLOT Y(X). Plot $Y(X)=Y1-Y2$ for each example. $Y(X)$ is the error function; show how it increases with X. What does this suggest about the use of Taylor polynomials for approximation?

Note: This would be a good lesson in which to consider doing the first example ($\sin x$) on-line, then each of the others as a separate compressed slide show.

Problem 20. Approaches to Function Approximation

Taylor's Theorem is not the only approach to polynomial approximation. Interpolation, Least-Squares, and Minimax are other important approaches. This lesson illustrates the different approaches by fitting straight lines to $y=\sin x$, $0 \leq x \leq \pi/2$.

Setup

Disk 1, select #4, PLOT Y1(X),Y2(X) SIMULTANEOUSLY.

(1) Set Y1(X)=SIN(X), Y2(X)=X, Xmin=0, Xmax=1.57, DELTA X=.03, scaling=MANUAL, Ymin=0, Ymax=1.57, screen=CLEAR, Y1=DOTTED, Y2=SOLID. Then PLOT. See the Taylor approximation straight line.

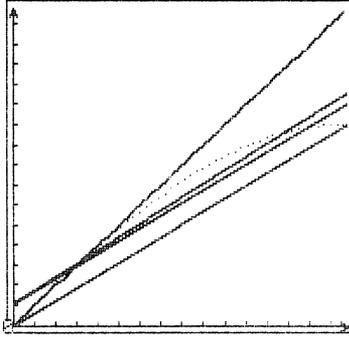
(2) Set Y1(X)=SIN(X), Y2(X)=.6366*X, DELTA X=1.57 (for speed), screen=KEEP. PLOT. See also the Interpolation line.

(3) Set Y1(X)=SIN(X), Y2(X)=.6644*X+.1148. PLOT. See also the Least-Squares line.

(4) Set Y1(X)=SIN(X), Y2(X)=.6366*X+.1053. PLOT. See also the Minimax line.

Display

```
ARBFPLOT4
Xmin=0
Xmax=1.57
Y1min=0
Y1max=1
Y2min=.105
Y2max=1.1
  ΔX=.1
  ΔY=.1
X=1.57
Y1=SIN(X)
Y2=.6366*X+.1053
```



Discussion

The Taylor line has the same value and slope as $y = \sin x$ at $x=0$. The Interpolation line passes through the points $(0,0)$ and $(\pi/2, 1)$, (slope= $2/\pi$). The Least-Squares line minimizes

$$\int_0^{\pi/2} (\sin x - (a+bx))^2 dx$$

The Minimax line minimizes the maximum error and thus lies halfway between the Interpolation line and the tangent line with the same slope.

Which approach is best? That depends on the information available and the purpose for the approximation.

If all we know about the sine function are its values at $x=0$ and $x=\pi/2$, then Interpolation gives us the best, actually the only, possible fit. If we want a straight line which "behaves like" the sine function for small x , i.e., has the same value and direction, Taylor is the best. If we want the "average" error to be as small as possible, then Least Squares is best. Least Squares is the proper model for many problems

involving experimental data. If we need to know the maximum error and wish to make that error as small as possible, as would be the case for a subroutine to be built into a calculator or programming language, then the Minimax approach is best.

Activities

(1) Plot $Y(X) = \text{SIN}(X) - (A \cdot X + B)$, where A and B are the coefficients for the four fits. $Y(X)$ is now the error function. For each fit, what is the maximum error? Where does it occur?

(2) Extend the comparison of methods by comparing the Taylor and Interpolation third-degree polynomials. To get the Interpolation cubic, pass a third-degree polynomial through the points $(X, \text{SIN}(X))$, for $X=0, \pi/6, \pi/3, \pi/2$. Compare the two fits by plotting them and by plotting the error functions.

Problem 21. Fourier Series

Fourier series can be used to approximate a discontinuous function, but a peculiar thing called Gibbs' phenomenon occurs near the discontinuity. We shall explore this graphically.

Note: This lesson is proposed to be a compressed slide show because the setup time for each graph is long, and the comparison of finished graphs is the important feature.

Setup

(1) Disk 1, select #1, PLOT Y(X).

Set $Y(X) = .785 * ((X < 3.141) - (X > 3.141))$, Xmin=0, Xmax=6.28, DELTA X=.033, scaling=MANUAL, Ymin=-1, Ymax=1, curve=DOTTED. Then PLOT.

After plotting, exit to the main menu. Save the display as a COMPRESSED GRAPH on your GRAPH PACK initialized diskette. Call your graph SQUARE WAVE.

(2) Return to ARBPLOT main menu. Select #4, PLOT Y1(X), Y2(X), SIMULTANEOUSLY.

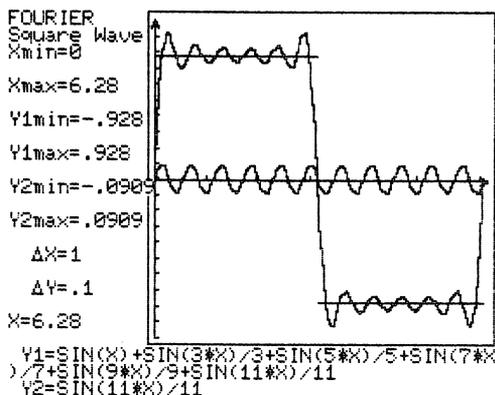
Set screen=KEEP, Xmin=0, Xmax=6.28, DELTA X=.1, scaling=MANUAL, Ymin=-1, Ymax=1, both curves SOLID, $Y1(X) = \text{SIN}(X) + \text{SIN}(3*X)/3 + \dots + \text{SIN}(n*X)/n$, $Y2(X) = \text{SIN}(n*X)/n$, where $n=3, 5, 7, 9, \text{ or } 11$. Then PLOT.

After each plot, exit to the main menu. Save the display as a COMPRESSED GRAPH. Call each successive approximation FIRST, SECOND, THIRD, etc.

From your GRAPH PACK diskette reload your SQUARE WAVE graph after saving each graph and repeat step (2) until you have built a set of compressed graphs for $n=3, 5, 7, 9, 11$.

(3) Make a COMPRESSED SLIDE SHOW of your graphs (see Section 2.5.3).

Display



Discussion

Each slide displays the most recent approximation and the contribution the last term makes to the approximation. Run through the slide show; Gibbs' phenomenon becomes evident.

Activities

Construct additional sequences of Fourier approximations for both discontinuous and continuous functions. For example, try discontinuous and continuous sawtooth patterns. Note that Gibbs' phenomenon occurs only for discontinuous functions.

Problem 22. Lissajous Figures

If we plot $x=\sin(a+bt)$ against $y=\sin(c+dt)$, we get a Lissajous figure. Lissajous figures are used in electronics as an aid in analyzing alternating currents of differing frequency and phase.

Setup

Disk 1, select #2, PLOT X(T) VS. Y(T).

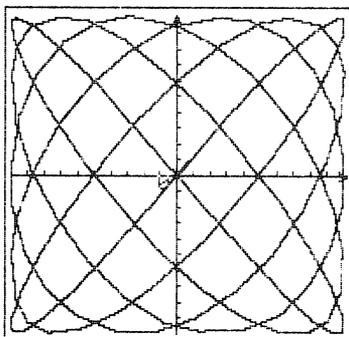
Set Tmin=0, Tmax=6.3, DELTA T=.05, scaling=AUTOMATIC and INDEPENDENT.

Set $X(T)=\sin(5*T)$, $Y(T)=\sin(6*T)$. Then PLOT.

Change to $X(T)=\sin(5*T)$, $Y(T)=\sin(7*T)$. PLOT.

Display

```
AREPLOT2
Tmin=0
Tmax=6.3
Xmin=-1
Xmax=.999
Ymin=-.996
Ymax=1
  ΔX=.1
  ΔY=.1
T=6.3
X =SIN(5*T)
Y =SIN(6*T)
```



Discussion

Lissajous figures are good examples to use to introduce parametric equations. The equations are simple yet the curves

are intricate and the curves have a clear physical interpretation.

The first curve has a period of 2π ; the second, π . How can you tell in advance what the period will be?

Activities

Construct some additional Lissajous figures, both simple and intricate such as:

$$X(T)=\text{SIN}(T), \quad Y(T)=\text{SIN}(2*T) \quad (\text{Bowtie})$$

$$X(T)=\text{SIN}(T), \quad Y(T)=\text{COS}(2*T) \quad (\text{Parabola})$$

$$X(T)=\text{SIN}(12*T), \quad Y(T)=\text{SIN}(13*T) \quad (\text{Pillow})$$

Use $\text{DELTA } T=.025$ for the last one. You might want to use **MANUAL** scaling to see your curve sooner.

Note: If you use Lissajous figures to introduce Parametric Equations you might follow a brief dynamic presentation with a slide show consisting of a variety of parametric curves.

Problem 23. Cycloids

The path followed by a point on the rim of a wheel rolling along a straight line is called a cycloid. Related curves are swept out by points interior to the wheel or on a flange, as on a railroad wheel. The parametric equations are

$$x = at - b \sin t, \quad y = a - b \cos t,$$

where t is the angle of rotation, a is the radius of the wheel, and b is the distance from the wheel's center to the moving point.

Setup

Disk 1, select #2, PLOT X(T) VS. Y(T).

(1) Set $X(T)=T-\text{SIN}(T)$, $Y(T)=1-\text{COS}(T)$, $T_{\min}=-2$, $T_{\max}=8$, $\text{DELTA } T=.1$, $\text{scaling}=\text{AUTOMATIC}$ and INDEPENDENT . Then PLOT.

(2) Change scaling to SAME. PLOT.

(3) Set $\text{screen}=\text{KEEP}$, $\text{scaling}=\text{MANUAL}$ and SAME but accept present values.

Set $X(T)=T-1.5*\text{SIN}(T)$, $Y(T)=1-1.5*\text{COS}(T)$. PLOT.

Set $X(T)=T-.5*\text{SIN}(T)$, $Y(T)=1-.5*\text{COS}(T)$. PLOT.

Display

AREPLOT2

Tmin=-2

Tmax=8

Xmin=-1.55

Xmax=7.51

Ymin=.5

Ymax=1.5

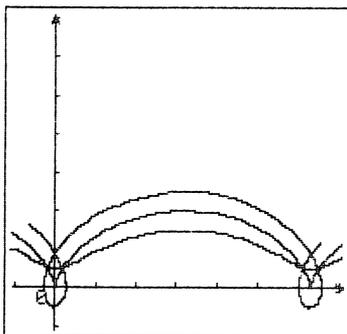
$\Delta X=1$

$\Delta Y=1$

T=8

$X = T-.5*\text{SIN}(T)$

$Y = 1-.5*\text{COS}(T)$



Discussion

The first curve surely doesn't look like a cycloid. When we are describing two-dimensional physical objects, it is usually a good idea to have scaling on both axes the same. Now look at the second curve. That's more like it. Superimposing the exterior and interior curves shows us what happens on the wheel. What is the path followed by a point on the tread of an under-inflated pneumatic tire?

Activities

The path followed by a point on the rim of a wheel rolling inside another wheel is called a hypocycloid. The equations are:

$$x(t) = (a-b)\cos t + b \cos\left(\frac{a-b}{b}t\right)$$

$$y(t) = (a-b)\sin t - b \sin\left(\frac{a-b}{b}t\right)$$

where a is the radius of the fixed circle, b is the radius of the rolling circle, and t is the angle of rotation of the center of the rolling circle. Construct hypocycloids for $b = a/3$, $a/4$, $a/6$, and $2a/9$.

How would the equations change for a circle rolling outside a fixed circle (called epicycloid)? Construct epicycloids with the same proportions as the hypocycloids.

Problem 24. Introducing Polar Coordinates

Properties of symmetry, extent, periodicity, and the many-one relationship of coordinates to points for curves in polar coordinates are different from the properties of curves in rectangular coordinates. Let's plot a succession of curves to show the properties.

Setup

Disk 1, select #3, PLOT R(T) (POLAR).

Set Tmin=0, Tmax=6.3, POINTS PER REVOLUTION=64,
scaling=MANUAL, Rmin=0, Rmax=2.

(1) Set $R(T)=1$. PLOT.

(2) Set $R(T)=1+\sin(T)$. PLOT.

(3) Set $R(T)=1-\cos(T)$. PLOT

Change to Rmin=0, Rmax=1.

(4) Set $R(T)=\sin(2*T)$. PLOT.

(5) Set $R(T)=\cos(2*T)$. PLOT

(6) Set $R(T)=-\sin(T)$. PLOT

(7) Set $R(T)=\cos(T)$. PLOT

Change to Tmin=-5, Tmax=12, POINTS PER REVOLUTION=32,
scaling=AUTOMATIC.

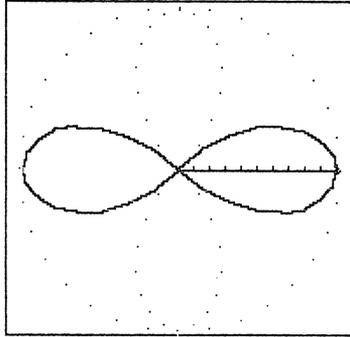
(8) Set $R(T)=T$. PLOT.

(9) Set $R(T)=\exp(.2*T)$. PLOT.

Display

```
ARBPLOTS
Tmin=0
Tmax=6.3
Rmin=-1
Rmax=1

ΔR=.1
T=6.28
R = COS(2*T)
```



Discussion

(1) We start by showing that the basic curve ($R=\text{const.}$) is a circle rather than a straight line ($Y=\text{const.}$).

(2), (3) The familiar sine and cosine curves become cardioids. Notice and discuss the symmetries of sine-only and cosine-only functions.

(4), (5) R can be negative. Again notice the symmetries.

(6), (7) As T goes from 0 to 2π , the curve is swept out twice. A point in the plane can have many coordinate pairs.

In (4) and (5) the period of the functions is π , but the period of the curve is 2π . In (6) and (7) the period of the functions is 2π but the period of the curves is π . What causes this phenomenon?

(8), (9) We see that polar coordinates have uses other than for periodic functions. The curves are the Archimedean spiral and the Spira Mirabilis or logarithmic spiral. Why logarithmic? What are physical conditions giving rise to these?

Activities

As a slide show and/or student assignments, explore other polar curves, e.g., limacons, roses, lemniscates, conchoids, and other spirals. Explore combinations, e.g., $r=a+b \cos kt$. See Problem 13 for polar equations for conic sections.

Problem 25. Polar Integration

We shall use computer graphics to show the basic increment of summation in finding area in polar coordinates and we shall show the process of summation.

Setup

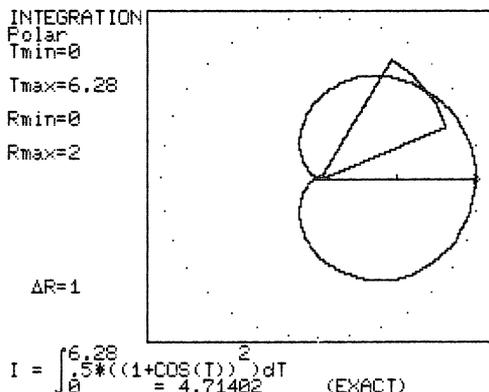
Disk 2, select #6, POLAR INTEGRATION.

Set $R1(T)=1+\cos(T)$, $R2(T)=0$, $Tmin=0$, $Tmax=6.284$, number of subdivisions=12, mode=TYPICAL ELEMENT AT $T=.6$, antiderivative $I(T)=.75*T+\sin(T)+\sin(2*T)/8$. Then INTEGRATE.

Change mode=SUM. Then INTEGRATE.

Change number of subdivisions=24. INTEGRATE.

Display



Discussion

We see first the typical element in integrating to find area in polar coordinates, next we approximate the area as the

sum of circular sectors, and then we increase the number of sectors to show approaching the true value.

Now use the examples in Section 3.5.4 to show area between two curves, area with negative radius, and area between two loops of the same curve.

Activities

Using the technique of the last example of Section 3.5.4, find the net area of the limaçon, $r=3+5 \sin t$ (outside area minus the area of the "hole").

Problem 26. Convergence and Summation of Alternating Series

Convergent alternating series have the desirable property that partial sums alternate between larger and smaller than the sum of the series, with the difference between sum and partial sum always less than the magnitude of the first term not used. Newcomers to infinite series frequently have difficulty visualizing this phenomenon. This lesson illustrates convergence of alternating series graphically and numerically.

Setup

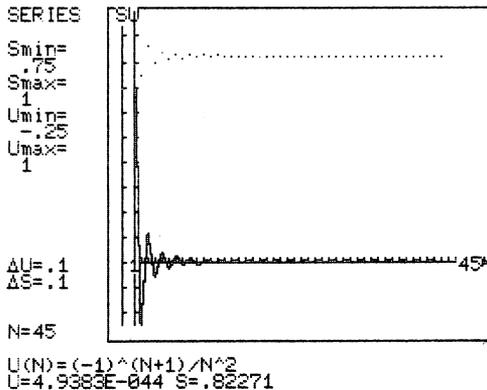
Disk 1, select #8, SEQUENCES AND SERIES.

(1) Set $U(N)=(-1)^{(N+1)}/N^2$, $F(N)=NONE$, starting value for $N=1$. Then PLOT. See oscillation of terms and partial sums.

(2) Set limits, starting value=1, number of terms=45. PLOT. See sum settle down to .822xx.

(3) Set limits to continue for 51 more terms. PLOT. See that .822 persists and oscillation is in the fourth and fifth decimal places.

Display



Discussion

How many terms are needed to calculate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ accurate to three decimal places?

$$\frac{1}{(n+1)^2} < .5 \cdot 10^{-3} \Rightarrow n > 44.$$

Step 1 shows the oscillation of terms and partial sums.

Step 2 shows that the sum is .822xxx.

Step 3 verifies that the first three decimal places do not change for $n > 45$. The sum to five decimal places is .82247.

Activities

Repeat the exercise for:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$$

Calculate the number of terms needed for three decimal place accuracy. Plot sufficient terms to show convergence. $N!$ must be defined recursively, so in the second case, set $U(N) = -U(N-1)/N$, with $U(1)$ preset to 1.

Problem 27. Direction Fields and Integral Curves

We shall describe the integral curves of the differential equation $dy/dx=-x/y$.

Setup

Disk 2, select #8, DIFF. EQUATIONS/DIRECTION FIELDS.

Set $DY/DX=-X/Y$, initial condition $X0=0$, $Y0=3$, mode=DIRECTION FIELD AND INTEGRAL CURVE, scaling=MANUAL, $Xmin=-4$, $Xmax=4$, $Ymin=-4$, $Ymax=4$, screen=CLEAR. Then PLOT.

Display

MH PLOT3

X0=0

Y0=3

$\Delta X=1$

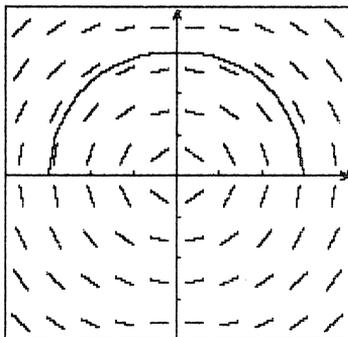
$\Delta Y=1$

Ymin =
.0952

Ymax =
3

X: [-4,4]
Y: [-4,4]

$dy/dx=-X/Y$



Discussion

The integral curves of $DY/DX=-X/Y$ are the circles $x^2+y^2=r^2$. This is easy to obtain directly since the differential equation is separable, but you can also verify this by using the fact that a circle's radius is perpendicular to the tangent it intersects. Note that the graph shows only

the upper half of a circle of radius 3; to get the lower half, set the initial condition to $X_0=0$, $Y_0=-3$, and PLOT again.

Activities

Describe the integral curves for $dy/dx=x/y$.

Problem 28. Logistic Population Models

The logistic population model (Appendix A) is $dy/dx=(a-by)y$. If $A=.5$, $B=.05$, we have $dy/dx=(.5-.05y)y$. We shall obtain integral curves representative of the different kinds of behavior of the solutions.

Setup

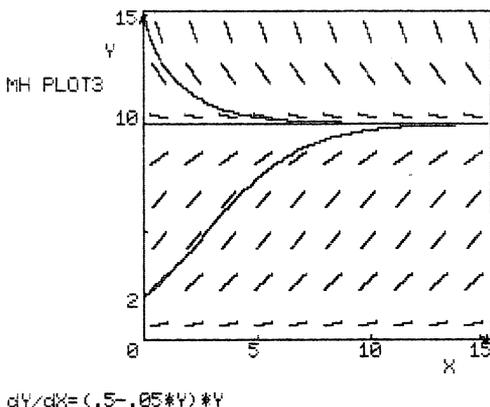
Disk 2, select #8, DIFF. EQUATIONS/DIRECTION FIELDS.

Set $DY/DX=(.5-.05*Y)*Y$, initial condition $X0=0$, $Y0=2$, mode=DIRECTION FIELD AND INTEGRAL CURVE, scaling=MANUAL, $Xmin=0$, $Xmax=15$, $Ymin=0$, $Ymax=15$, screen=CLEAR. Then PLOT.

Change initial condition to $X0=0$, $Y0=15$, mode=INTEGRAL CURVE, screen=KEEP. Then PLOT.

Change initial condition to $X0=0$, $Y0=10$. PLOT again.

Display



Discussion

The last curve shows a steady-state of $Y=10$. If the initial population is $Y=2$ (the first curve), the population grows rapidly at first, then slows as it approaches a "carrying capacity" of 10. If the population starts above the carrying capacity, the population decreases asymptotically to it. Also note that the population grows fastest at $Y=5$ (the maximum of $(.5-.05y)y$); this indicates the level for harvesting a population to obtain the greatest yield per unit time.

Activities

(1) A steady-state population is a population which is approached asymptotically as x increases. An equilibrium state is a state which if entered is never left. In this example there are two equilibrium states and one steady state. What are they?

(2) Answer (1) for the general model $dy/dx=(a-by)y$. At what population level is the population increasing the fastest?

(3) Suppose I want to raise catfish in a pond and that the population satisfies the general logistic differential equation. What is the best strategy for harvesting the catfish?

(4) Newton's law of cooling: Suppose a room or other environment has a constant temperature T_0 and a relatively small object of different temperature is left to cool (or warm). If T is the temperature of the object at time t , then it is reasonable to assume that the rate at which the object changes temperature is proportional to the difference in temperature between the environment and the object, i.e., $dT/dt = k(T_0-T)$. Assuming that $k=.01$ and $T_0=70$, draw a direction field. Draw integral curves through the initial points $(0,30)$, $(0,70)$ and $(0,210)$ and interpret your graph.

Problem 29. Orthogonal Trajectories

Show how the orthogonal trajectories of a family of curves can be obtained through manipulation of differential equations.

Setup

Disk 2, select #8, DIFF. EQUATIONS/DIRECTION FIELDS.

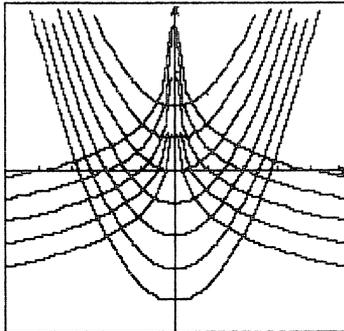
Set $DY/DX=X$, initial condition $X0=0$, $Y0=-4$, mode=INTEGRAL CURVE, scaling=MANUAL, $Xmin=-5$, $Xmax=5$, $Ymin=-5$, $Ymax=5$, screen=CLEAR. Then PLOT.

Change screen to KEEP, initial condition $Y0=-3$ ($X0$ stays fixed), and PLOT again. Continue increasing $Y0$ and plotting until $Y0=3$.

Now change the differential equation to $DY/DX=-1/X$, initial condition $X0=-4$, $Y0=0$, and PLOT. Hold $Y0$ fixed while increasing $X0$ by one and plotting until $X0=4$.

Display

```
MH PLOT3
X: [-5,5]
Y: [-5,5]
ΔX=1
ΔY=1
```



The parabolas are integral curves of the d.e. $dy/dx = x$, and the other curves are the orthogonal family given by $dy/dx = -1/x$.

(Text changes made with MH GRAPH EDITOR)

Discussion

The direction lines of the two differential equations (if they were drawn) are orthogonal, since $X \cdot (-1/X) = -1$. The resulting curves form "orthogonal trajectories" since any two intersecting curves are orthogonal (i.e., their tangents are orthogonal) at points of intersection. If we had started with a one-parameter family of curves rather than the differential equation, we could at times have found the orthogonal family by first finding a differential equation $dy/dx=f(x,y)$ determining the first family, then solving $dy/dx=-1/f(x,y)$ to find the second family. For example, starting with the family $y=x^2+C$, we could differentiate each side to obtain $dy/dx=2x$, then solve $dy/dx=-1/2x$ for the orthogonal curves.

Problem 30. Exploring

One of the best uses of the computer by the student or class is to explore new territory, to answer "what if" questions. Analytic geometry, particularly parametric equations and polar coordinates, can be used to produce many interesting figures. Try developing some of your own.

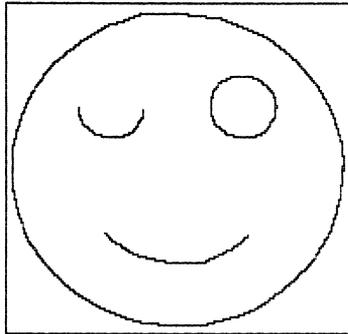
Setup

How was the following graph prepared? (Hint: The same functions were used throughout.)

Display

```
ARBPLOT2
Tmin=0
Tmax=6.3
Xmin=-1
Xmax=1
Ymin=-1
Ymax=1

T=6.3
X =COS(T)
Y =SIN(T)
```



Discussion/Activities

You can start either with (1) how was the figure constructed, or (2) what is the curve of the equation, and go as far as you please.

6. Technical Notes

ARBPLOT consists of sixteen applications programs; four programs used in saving and modifying displays and running slide shows; and one binary file containing several machine language subroutines, shape tables, and high-res character sets. The package requires a 48K Apple II Plus computer with one disk drive, or a 48K Apple II with Applesoft ROM or Language System and one disk drive, each with DOS 3.3. A printer with graphics capability can be used by the package.

Because most of the programs are large (50-60 sectors in their original form), it was necessary to "crunch" the programs to make the package fit on two diskettes. The crunching process removes all REMarks and unnecessary line numbers, saving considerable space and running time, but also making the programs unreadable. In order to help those who would like an understanding of the working of the programs, Appendix C has a listing of the uncrunched version of ARBPLOT1. For those who wish to incorporate another printer into the SAVE DISPLAY process, Appendix D contains a listing of DISPLAY.

The major design considerations for the programs were:

1. Modularity for ease of adding new programs
2. Menu driven for ease of use
3. Complete input control
4. Complete error trapping
5. Function decoding
6. Scaling and windowing in curve plotting

7. Printing on the high-resolution graphics screen
8. Pointers to improve speed in critical portions

These considerations will be discussed using ARBPLOT1 as an example. ARBPLOT1 was the basic building block for the ARBxxxx programs; the principles are the same but the execution is different for the MH PLOT programs.

The ARBPLOT disks have been formatted with a "nine-descending sector skewing" which results in about a 65% improvement in program loading time. The method is described in D. Worth and P. Lechner, "The Speed Sector," Softalk, April 1982.

6.1. ARBxxxx Programs

All ARBxxxx programs are organized in the same way. All use high-resolution graphics page 2 (\$4000-\$5FFF) and a common binary file called ARBPCHAR residing at 24576-27647 (\$6000-\$6BFF). LOMEM is set to 27648 (\$6C00) to place all variables above the binary file. The programs are organized as follows:

<u>Line No.</u>	<u>Function</u>
1	Set LOMEM; jump to introduction
2-4	Function definitions
100-610	Miscellaneous small subroutines
800-890	Introduction; calls SR24000
1000-1190	Master menu
2000-2990	Change function; calls SR18700
3000-3990	Change limits and scaling
4000-4290, 4500-4990	Process menu choices 3-5
4300	Recall display
4400-4410	Exit to ARBPLOT menu
5000-5990	Calculate and plot; calls SR18000, 18100, 18220
6000-8990	Special calculations and plotting

5870-5900	End of plot processing; return
or	to menu
9000-9500	
18000-18090	Evaluate functions,
	find true min and max
18100-18210	Plot curve - automatic scaling
18220-18590	Plot curve - manual scaling
18600-18630	Print three significant figures
18700-18880	Decode function
19600-19630	Case converter for hi-res text
20000-21090	General input routine
21100-22390	Error processing
24000-24300	Initialize variables,
	load ARBPCHAR

ARBPCHAR contains seven files:

<u>Name</u>	<u>Location</u>
Hi-res character generator	\$6000.\$100 24576
MH data	\$6100.\$300 24832
Tiger graphics	\$6500.\$100 25856
ARB data	\$6600.\$0F0 26112
Error handling routine	\$66F0.\$00F 26352
SINTBL	\$6700.\$100 26368
Hi-res character table	\$6800.\$400 26624

ARBPLOT1 illustrates typical content and use of the modules. The introductory message appears while the variables are initialized, before the master menu (lines 1000-1190) appears. All actions return to the master menu. Subsidiary menus occur where needed; none are in ARBPLOT1. Any action except PLOT is reversible either by an "OKAY(Y/N)?" question or by repeating the selection from the menu in those cases where two choices "toggle" from one to the other. You can return from PLOT; however, you must first complete the process. All two-way actions are toggled; see lines 4000-4210.

Input control is handled in two ways. For most single character responses, GET A\$ is used, with rejection of unwanted responses (lines 1160-1190, 500-520). For multiple character response, a general purpose input routine, ARBREADER, is used with supplemental response checking (lines 20000-21090, 300-

310). ARBREADER has many features not used in this package, such as multiple fields per line and "HELP" responses. The major features are described in Appendix F.

Error trapping has for its objective detecting and responding to any Applesoft error under program control. In the ARBPLOT programs, input errors are avoided by input control, and output errors are avoided by checking all output (primarily potential off-scale graphics) before printing or plotting (see use of F3 in SR18220), leaving only problems of function definition and evaluation. These are handled by ONERR GOTO 21100 and lines 21110-21170.

Function decoding modifies the program during execution to create a DEF FN statement as the second statement in the program. First, you must find the memory address of the beginning of the second line of the program. Lines 24090 and 24180 do this. Next, you must decode the character string defining the function and POKE it into that line. Functions, arithmetic and logical operations, and logical relations must be tokenized (see page 121, Applesoft II Reference Manual). SR18700 does this, using the token table built in lines 24210-24240. Finally, the result must be checked to be sure a legal DEF FN statement has been created, and the statement must be activated. The decoding subroutine does part of the checking; the process is completed in line 2070, which activates the function, backed up by lines 21127, 21300, and 22200-22220, which check for errors in definition.

Drawing the curve properly is handled in two ways. If automatic scaling is used, SR18000 calculates an array of function values and finds minimum and maximum values. Lines 5030-

5240 convert the Ymin and Ymax into scale factors so that SR18100 can plot the curve. If manual scaling is used, you set Ymin and Ymax, then SR18220 checks each point computed to plot, reject, or interpolate across the boundary of the plot window.

Printing on the high-resolution graphics screen is easy using the high-resolution graphics text routines by Christopher Espinosa from the Apple Software Bank Contributed Programs, Volume 3. The program and character set are in ARBPCHAR. Activating high-resolution print requires only POKE 54,0: POKE 55,96 (line 5300); returning to text mode requires only PR#0: TEXT (line 5900). Espinosa's character set includes lower case, provision for changing character definition, and provision for alternative character sets. SR19600 handles case conversion for ARBPLOT. You will notice several special characters on the screen, and MH GRAPH EDITOR has an alternative character set.

Note on Applesoft graphics: High-resolution shapes can be handled in three ways on the Apple:

- (1) HPLOTting points and lines through BASIC,
- (2) shape tables, and
- (3) moving blocks of bytes.

All three of these methods have been used in ARBPLOT. The HPLOT is the basis for most of the curve-plotting. Shape tables are used, for example, for the tangent lines in MH PLOT1 and the "darts" in ARBINT1; the XDRAW instruction allows these lines to be "undrawn" without disturbing the original graph. The shape tables for these lines and forms are contained within the ARBPCHAR file. Block transfers are the basis for Christopher Espinosa's High-Resolution Character Generator which is used in most of the ARBPLOT programs.

Long programs in Applesoft tend to be slow. The farther into the program you go, the slower it is. It is possible to fool Applesoft into thinking the beginning of a subroutine is the beginning of the program by moving pointers. CALL 24832 does this, activated from lines 18000, 18100, and 18220, then deactivated in lines 18080, 18210, and 18580. Moving pointers is risky and can cause a program to self-destruct.

Generally, programming tricks should be avoided unless they make major program improvements that can be made in no other way. Tricks make programs harder to understand, harder to transport to other computers, and tend to make programs less reliable. We believe that the tricks used in the ARBxxxx and MHxxxx programs do improve the programs significantly and are worth the disadvantages. If you wish to learn more about Applesoft, its inner workings, and how to beat the system, read All About Applesoft published by A.P.P.L.E. (Apple Pugetsound Program Library Exchange) and their magazine, Call A.P.P.L.E., available at many computer stores or directly from A.P.P.L.E., 304 Main Avenue South, Suite 300, Renton WA 98055.

6.2. MH PLOTxx Programs

The MH PLOT1 - MH PLOT3 programs use the same basic structure as that described for ARBxxxx programs. One difference is that the line-numbering is not uniform; however, the major blocks (input routine, function decoding, etc.) are easy to find. Another difference is the absence of a case-conversion routine for the High-Resolution Character Generator. This was not necessary because the programs were written on an Apple

equipped with a Lower Case Adapter (LCA) chip. Text that is entered directly as lower case will list as strange symbols on an Apple without lower case display, but this will not affect the successful display of lower case in the high-resolution graphics mode. LCA chips are commercially available for under \$30 (e.g., from A.P.P.L.E., Renton, WA) and are useful in many applications, notably word processing.

6.3. Display Saving and Slide Show Programs

Four programs are involved in the display saving and slide show processes: DISPLAY, SHOW, GRAPH PACK and MH GRAPH EDITOR. All are reached through option 9 in the ARBPLOT menu; their functions are described in Section 2.5.

You may wish to change DISPLAY to incorporate another printer. The program listing for DISPLAY is given in Appendix D. Printer control is in lines 3000-3800, with the Apple Silentype in lines 3200-3270, IDS440 Paper Tiger in lines 3400-3510, and MX-80 with Grappler controller in lines 3600-3650. If your printer driver is in BASIC, you could put it in lines 3800-3990, ending it with GOTO 100 to return to the DISPLAY menu. If it uses a machine language subroutine, you might be able to place it in one of the gaps in ARBPCHAR. If incorporating another printer enlarges the DISPLAY program, you can only have it on ARBPLOT Disk 2 (Disk 1 is full).

GRAPH PACK is the one program in the package that is not quite what it seems. The Applesoft end-of-program pointer has been moved to allow incorporation of three binary files within the program. These files will not show up in a LIST, but they

are very much there -- changing the program will destroy it if not done with careful consideration of Applesoft pointers. This abuse of programming clarity was used for two reasons:

- (1) the program loads faster than if the files were loaded separately, and
- (2) GRAPH PACK was written to be moved to other diskettes (on which slides are saved). Since all supporting binary files are contained within GRAPH PACK, there is no danger of separating them.

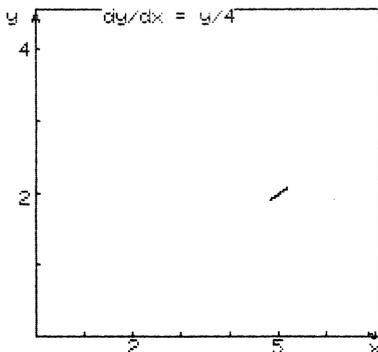
The theory and practice of compressing graphs can be found in Mark Harris, "An Apple Slide Show," Creative Computing, February 1982.

Appendix A

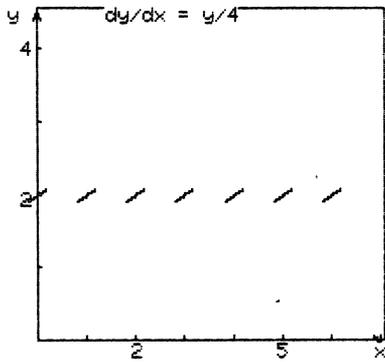
A Geometric Approach to First-Order Differential Equations: Single-Species Population Models

Suppose we wish to model the growth of a population of some species, say rabbits. If the rabbits are given lots of space and food, it is reasonable to assume that the rate of growth of the population is proportional to the population itself. Letting $y(x)$ be the number of rabbits at time x , we have the differential equation $dy/dx=ky$ for some constant k . Assuming for the time being that $k=1/4$, we shall graph the solution curve y vs. x . Naturally, the curve will depend on the initial number of rabbits, $y(0)$. Without this information, the best we can do is to find a family of such curves (called integral curves), one for each possible value of $y(0)$. We will do this by means of a direction field.

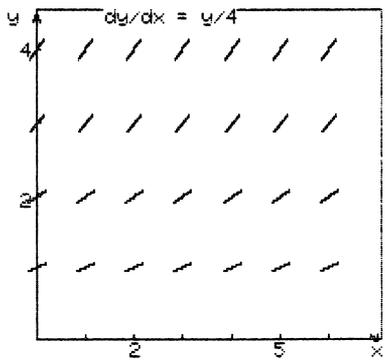
Suppose a solution curve passes through the point $(5,2)$. Then the differential equation $dy/dx=(1/4)y$ tells us that the slope of the curve at this point is $dy/dx=(1/4)2=1/2$. To indicate this, draw a small "direction line" of slope $1/2$ at this point as follows:



Since for this differential equation the slope depends only on the y-coordinate of the point, we may duplicate this direction line along the line $y=2$:

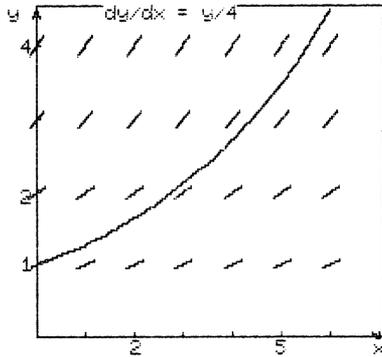


If we continue drawing direction lines throughout the graph (we are concerned only with the upper half in this example, since $y < 0$ is bad news for the rabbits), we get the following direction field:



What does it all mean? The field shows the direction that a solution curve must follow as it meanders through the plane, for all possible starting points. For example, here is a

solution curve with initial condition $y(0)=1:7$



This approach to solving differential equations geometrically will work for any differential equation of the form $dy/dx=f(x,y)$. (If the function on the right depends on x alone, then the solutions are just the antiderivatives of the function.)

Let us now consider a more sophisticated population model. Suppose that the rabbits share a fixed food supply. Then in the differential equation $dy/dx=ky$, we could replace the constant k with a function $k(y)$ that decreases as y increases. This would reflect a decreasing net birth rate: the more rabbits there are, the less the chances of a given newborn rabbit surviving because of the increasing scarcity of food. The most simple function $k(y)$ is a linear function $k=a-by$. (This is called the logistic model.)

T

Appendix B Richardson Extrapolation

Let $I = \int_a^b f(x) dx,$

$$T_N = h \left(\frac{f_0}{2} + f_1 + f_2 + \dots + f_{N-1} + \frac{f_N}{2} \right),$$

$$\text{where } h = \frac{b-a}{N}$$

The error is given by

$$E_N = I - T_N = - \frac{(b-a)^2}{12} h^2 f''(z), \quad a < z < b$$

$$\text{or } T_N = I + O(h^2).$$

It can be shown that $f(x)$ having a continuous derivative of order 4, $a < x < b$, implies

$$T_N = I + Ah^2 + O(h^4),$$

where A is a constant independent of h .

Then

$$T_N = I + Ah^2 + O(h^4)$$

$$T_{2N} = I + Ah^2/4 + O(h^4).$$

Eliminate A , solve for I .

$$I = \frac{4T_{2N} - T_N}{3} + O(h^4)$$

Richardson Extrapolation thus starts with a Trapezoidal Rule approximation of N subdivisions, calculates the trapezoidal approximation for $2N$ subdivisions, and combines them as

$$R = \frac{4T_{2N} - T_N}{3}$$

to give an approximation of order 4.

Appendix C

ARBPLOT1 Listing

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1 LOMEM: 27648: GOTO 800: REM ..ARBPLOT1, VERSION2.1..SEE LINE 30123
2 DEF FN Y(X) = X * ((X - 1) / 2) : 2: RETURN : REM .....
100 REM ..FLÄGS
101 REM F1:SCREEN, F3:MANUAL, F5:CURVE, F6:PLOT, F7:SCALING, F8:AXES
228 :
229 REM ..CLEAR A LINE ON SCREEN
230 FOR I = K1 TO 5: PRINT B$:: NEXT : PRINT : RETURN
238 :
239 REM ..CHECK FOR NUMERIC ANSWER
300 F = K1: IF R1(K1) < > K1 THEN F = K2: PRINT "ANSWER MUST BE NUMERIC."
310 RETURN
398 :
399 REM ..SET HGR2 W/O CLEARING
400 POKE - 16304,K0: POKE - 16302,K0: POKE - 16299,K0: POKE - 16297,K0: HCOLOR=
      3: POKE 54,K0: POKE 55,96: PRINT : RETURN
498 :
499 REM ..OKAY
500 VTAB T: HTAB 16: PRINT "OKAY (Y/N)?": GET A$: HTAB K1: IF A$ = "Y" THEN F = K
      2: RETURN
510 IF A$ = "N" THEN F = 3: RETURN
520 PRINT CHR$(7): CHR$(7): GOTO 500
798 :
799 REM ..INTRODUCTION
800 ONERR GOTO 21100
810 POKE - 16368,K0: HOME : VTAB 4: HTAB 16: PRINT "ARBPLOT1"
820 PRINT : HTAB 9: PRINT "RECTANGULAR COORDINATES"
830 PRINT : HTAB 19: PRINT "BY"
840 PRINT : HTAB 12: PRINT "A. R. BROWN JR."
850 PRINT : PRINT "ARBPLOT1 PLOTS THE GRAPH OF ANY"
860 PRINT "FUNCTION Y OF A SINGLE REAL VARIABLE,"
870 PRINT "X, IN RECTANGULAR COORDINATES."
880 VTAB 23: HTAB 10: PRINT "PLEASE STAND BY...": GOSUB 24000
998 :
999 REM ..MAIN MENU
1000 HOME : PRINT ">>ARBPLOT1 - CURRENT VALUES & CHOICES<<"
1010 PRINT : PRINT "1. FUNCTION"
1020 PRINT "Y(X)=";Y$
1040 PRINT : PRINT "#2. LIMITS/SCALING": HTAB 22: PRINT "DELTA X=";DT
1050 FOR X = K1 TO 3 STEP K2
1060 HTAB 5:T = V1(X)
1065 PRINT RIGHT$(T$(X),5):: GOSUB 18600
1070 VTAB 7 + X / K2: HTAB 22:T = V1(X + K1)
1075 PRINT RIGHT$(T$(X + K1),5):: GOSUB 18600
1080 NEXT X
1090 HTAB 4: PRINT "SCALING::": HTAB 14: PRINT LA$(F7 + K2)
1100 PRINT : PRINT "3. SCREEN: ";LA$(F1):" PRESENT DISPLAY"
1110 PRINT "4. CURVE: ";LA$(F5 + 6):" LINE"
1120 PRINT "5. AXES: ";LA$(F8 + 8)
1130 PRINT : PRINT "ACTIONS::": PRINT "6. SHOW CURRENT DISPLAY"
1140 PRINT "7. PLOT CURVE": PRINT "8. EXIT"
1150 VTAB 22: PRINT "CHOICE: CHANGE VALUE(1-5)": HTAB 9: PRINT "ACT(6-8)"
1160 VTAB 24: HTAB 26: PRINT "WHICH(1-8)?": GET A$: F = ASC (A$) - 48: POKE - 16
      368,0
1180 IF F < K1 OR F > 8 THEN VTAB 24: HTAB K3: PRINT "ANSWER 1-8": CALL - 868: GOTO
      1160
1189 REM ..SELECT ACTION
1190 ON F GOTO 2000,3000,4000,4100,4200,4300,5000,4400
1998 :
1999 REM ..CHANGE FUNCTION
2000 HOME : HTAB 11: PRINT "-- FUNCTION --"
2010 VTAB 3: CALL - 958: PRINT "PRESENT CHOICE IS: PRINT " Y(X)=";Y$
2020 T = 6: GOSUB 500
2030 ON F GOTO 2020,1000,2050
2039 REM ..SET FUNCTION
2040 VTAB 8: CALL - 958
2050 VTAB 8: PRINT "ENTER THE FUNCTION TO BE PLOTTED."
2060 PRINT : PRINT "Y(X) = ": F = K0: GOSUB 18700:Y$ = R$(K1): IF F = K1 THEN 2050

2070 GOSUB 2:Y = FN Y(K1)
2100 GOTO 2000
2998 :
2999 REM ..CHANGE LIMITS & SCALING
3000 HOME : HTAB 8: PRINT "-- LIMITS & SCALING --"
3010 VTAB 3: CALL - 958: PRINT "X LIMITS: PRESENT LIMITS ARE"
3020 HTAB 5: PRINT "XMIN: ": T = V1(K1): GOSUB 18600: VTAB 4: HTAB 22: PRINT "XMAX
      = ": T = V1(K2): GOSUB 18600
3030 HTAB 5: PRINT "DELTA X=";T: DT: GOSUB 18600
3040 T = 7: GOSUB 500

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3050 ON F GOTO 3040,3210,3060
3060 VTAB 8
3070 PRINT "WHAT IS MINIMUM VALUE OF X?":: CALL - 958: GOSUB 20000
3075 IF R1(R1) = 7 THEN 3100
3080 GOSUB 300: ON F GOTO 3090,3070
3090 PRINT: PRINT "WHAT IS MAXIMUM VALUE OF X?":: GOSUB 20000
3100 PRINT: PRINT "WHAT IS MAXIMUM VALUE OF X?":: GOSUB 20000
3105 IF R1(R1) = 7 THEN 3130
3110 GOSUB 300: ON F GOTO 3120,3100
3115 T2 = (R1(R1):V1(K2)) = T2
3120 PRINT "MAXIMUM VALUE OF X MUST BE GREATER THAN XMIN.": GOTO 3070
3130 TM = R6 * (ABS(T1) + ABS(T2))
3135 PRINT: PRINT "WHAT IS INCREMENT (DELTA X)?":: GOSUB 20000
3140 PRINT: PRINT "WHAT IS INCREMENT (DELTA X)?":: GOSUB 20000
3155 IF R1(R1) = 7 THEN 3180
3160 GOSUB 300: ON F GOTO 3170,3150
3170 DT = R(R1)
3175 GOSUB 300: ON F GOTO 3170,3150
3180 PRINT "DELTA X MUST BE POSITIVE.": GOTO 3150
3190 IF T2 < 200 THEN PRINT "DELTA X MUST BE SUCH THAT AT MOST 200": PRINT "POINTS ARE PLOTTED.": GOTO 3150
3200 PRINT "POINTS ARE PLOTTED.": GOTO 3150
3210 VTAB 7: CALL - 958: PRINT "PRESENT SCALING IS ":LA$(F7 + K2):".":T = 9: GOSUB 500
3220 ON F GOTO 3210,3240,3230
3230 FT = F7 - F7: GOTO 3210
3240 IF F7 = R1 THEN 1000
3250 VTAB 9: CALL - 958: PRINT "SCREEN LIMITS ARE "
3270 VTAB 5:T = V1(3)
3275 PRINT "RIGHTS $(T$(3),5):": GOSUB 18600
3280 VTAB 10: HTAB 22:T = V1(4)
3285 PRINT "RIGHTS $(T$(4),5):": GOSUB 18600
3290 T = 12: GOSUB 500
3300 T = 12: GOTO 3250,1000,3315
3315 VTAB 12
3320 PRINT "WHAT IS MINIMUM VALUE OF Y?":: GOSUB 20000
3325 IF R1(R1) = 7 THEN 3420
3330 GOSUB 300: ON F GOTO 3410,3320
3400 Y4 = R1(1):V1(3)
3410 PRINT "WHAT IS MAXIMUM VALUE OF Y?":: GOSUB 20000
3420 Y4 = Y4
3430 PRINT "WHAT IS MAXIMUM VALUE OF Y?":: GOSUB 20000
3435 IF R1(R1) = 7 THEN 3440,3420
3440 V1(4) = R(R1)
3450 IF V1(4) < Y4 THEN PRINT "YMAX MUST BE GREATER THAN YMIN.": GOTO 3315
3460 GOTO 3250
3498 REM .TOGGLE KEEP OR CLEAR DISPLAY
4000 F1 = 3 - F1: VTAB 11: HTAB K1: PRINT "3. SCREEN ":LA$(F1):" PRESENT DISPLAY "
4010 GOTO 1160
4098 REM .TOGGLE SOLID OR DOTTED LINE
4099 F5 = 3 - F5: VTAB 12: HTAB K1: PRINT "4. CURVE ":LA$(F5 + 6):" LINE "
4110 GOTO 1160
4198 REM .TOGGLE SHOW OR DON'T SHOW AXES
4200 F8 = 3 - F8: VTAB 13: HTAB K1: PRINT "5. AXES ":LA$(F8 + 8):
4210 GOTO 1160
4298 REM .RECALL DISPLAY
4300 GOSUB 400: GOTO 5870
4310 REM .EXIT TO ARBPLOT MENU
4400 HOME: VTAB 10: HTAB 13: PRINT "CONFIRM EXIT.":T = 12: GOSUB 500: ON F GOTO 4410
4410 PRINT: PRINT D$: "RUNAPMENU"
4498 REM .EVALUATE FUNCTION, FIND TRUE MIN & MAX
4500 PRINT: Y1 = 1.237:Z = - Y1
4510 IF F7 = K2 THEN 5200
4520 FOR I = 3 TO N:V1(I) = V(I): NEXT
4530 REM .SET SCALE FACTORS
4540 Y4 = V1(3):Y5 = V1(4):I3 = (Y5 - Y4) / (T2 - T1) / C2
4550 IF Y4 < K0 THEN V1(4) = K0: GOTO 5200
4560 V1(3) = - K1:V1(4) = K1: GOTO 5200
4598 REM .SET GRAPHICS, HIRSH PRINTING
5300 F6 = K1: IF F1 = K2 THEN HGHT = 3: POKR 54,K0: POKR 55,96: GOTO 5340

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5310 GOSUB 400
5340 POKE 33,40: FOR X = 3 TO 13 STEP K2: VTAB X: PRINT B1$: NEXT
5348 :
5349 REM ..PLOT BORDERS AXES, TICS
5350 VTAB K1: PRINT "ABPLOT1 "
5360 HPLLOT 80,K0 TO 279,K0 TO 279,158 TO 80,158 TO 80,K0
5370 IF F8 = K2 THEN 5700
5380 Y = C4 + Y4 / Y3: IF Y > K1 AND Y < 158 THEN HPLLOT 85,Y TO 274,Y: HPLLOT 271,Y
+ K2 TO 275,Y TO 271,Y - K2
5390 X = C3 - T1 / T3: IF X > 79 AND X < 278 THEN HPLLOT X,C4 TO X,5: HPLLOT X - K2,
8 TO X,4 TO X + K2,8
5400 IF Y < K2 THEN Y = K2
5410 IF Y > 158 THEN Y = 158
5420 IF X < 80 THEN X = 80
5430 IF X > 277 THEN X = 277
5440 I = (T2 - T1) / K2
5450 E = K3 ^ INT (K4 ^ LOG (.95 ^ I)):L1 = INT (T1 / E) ^ E:L2 = INT (T2 / E) ^ E
5460 FOR I = L1 TO L2 + K6 ^ E STEP E
5470 IF I < T1 OR I > T2 THEN 5490
5480 XO = C3 + (I - T1) / T3: HPLLOT XO,Y TO XO,Y - K2
5490 NEXT
5500 VTAB 15: PRINT B1$;: HTAB 3: PRINT CHR$(2);"X=";E
5510 I = (V1(4) - Y4) / K2
5520 E = K3 ^ INT (K4 ^ LOG (.95 ^ I)):L1 = INT (Y4 / E) ^ E:L2 = INT (V1(4) /
E) ^ E
5530 FOR I = L1 TO L2 + K6 ^ E STEP E
5540 IF I < Y4 OR I > V1(4) THEN 5560
5550 YO = C4 - (I - Y4) / Y3: HPLLOT X,YO TO X + K2,YO
5560 NEXT
5570 VTAB 17: PRINT B1$;: HTAB 3: PRINT CHR$(2);"Y=";E
5580 X = INT (X / 7):Y = INT ((Y + 4) / 8) + K1
5590 IF X > 11 AND X < 40 AND Y > K0 AND Y < 21 THEN VTAB Y: HTAB X: PRINT K0
5598 :
5699 REM ..LABELS & CURVE
5700 VTAB 21: GOSUB 230
5710 VTAB 21: PRINT " Y =" ;Y$
5720 VTAB 23: GOSUB 230
5770 VTAB 19: PRINT "X="
5780 TM = K6 * ( ABS (T1) + ABS (T2))
5790 ON F7 GOSUB 18100,18220
5800 V(K1) = T1:V(K2) = T2
5810 FOR Y = K2 TO 4 STEP K2:TM = K6 * ( ABS (V(Y - K1)) + ABS (V(Y)))
5820 FOR X = K1 TO K2:XO = Y + X - K2
5830 T = V(XO): IF ABS (T) < TM THEN T = K0
5840 VTAB K2 # XO + K1: PRINT B1$
5850 VTAB K2 # XO + K1:A$ = T$(XO): GOSUB 19600: GOSUB 18600
5860 NEXT : NEXT
5868 :
5869 REM ..END PROCESSING
5870 POKE 973,255: POKE 33,40
5880 VTAB 24: HTAB 35:A$ = " #END ": GOSUB 19600: PRINT
5890 F6 = K0
5900 GET A$: POKE 54,K0: POKE 55,96: POKE 973,K0: VTAB 24: HTAB 32: PRINT B$: PR#
17996 :
17997 : REM >>>MAJOR PROCESSING SUBROUTINES<<<
17998 :
17999 REM ..EVALUATE FUNCTION, FIND TRUE MIN & MAX
18000 CALL 24832
18010 I = K0: FOR T = T1 TO T2 + K6 * DT STEP DT:Y = FN Y(T)
18040 IF Y < Y1 THEN Y1 = Y
18050 IF Y > Y2 THEN Y2 = Y
18060 Y(I) = Y:I = I + K1
18070 NEXT :V(3) = Y1:V(4) = Y2
18080 POKE 103,P1: POKE 104,P2: RETURN
18098 :
18099 REM ..PLOT CURVE - AUTOMATIC SCALING
18100 CALL 24832:R1 = C3 - T1 / T3:R2 = C4 + Y4 / Y3
18102 I = K0:X = T1:Y = Y(K0)
18104 XO = X / T3 + R1:YO = R2 - Y / Y3
18106 HPLLOT XO,YO
18110 FOR T = T1 TO T2 + K6 * DT STEP DT
18120 X = T:Y = Y(T):I = I + K1
18130 IF ABS (T) < TM THEN T = K0
18140 VTAB 19: HTAB 3: PRINT B$;: HTAB 3: GOSUB 18600
18150 X = X / T3 + R1:Y = R2 - Y / Y3
18160 IF F5 = K2 THEN HPLLOT XO,YO TO X,Y:XO = X:YO = Y
18170 IF F5 = K1 THEN HPLLOT X,Y

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18180 IF PEEK (C7) < C6 THEN NEXT : GOTO 18210
18190 POKE - 16368,KO: POKE 54,KO: POKE 55,96: GET A$: IF A$ = CHR$(5) THEN 182
10
18200 NEXT
18210 POKE 103,P1: POKE 104,P2: RETURN
18218 :
18219 REM ..PLOT CURVE - MANUAL SCALING
18220 CALL 24832:R1 = C3 - T1 / T3:R2 = C4 + Y4 / Y3
18230 F = 4: FOR T = T1 TO T2 + K6 * DT STEP DT:F3 = KO:Y = FN Y(T): IF Y < Y4 OR
Y > Y5 THEN F3 = K1
18231 REM .F5:1=DOTTED,2=SOLID
18232 REM .F3:0=GOOD,1=OFFSCALE,4=ERROR
18233 REM .F=OLD F3
18234 REM ..IF ERROR F3=4 SET IN LINE 21150
18240 IF ABS (T) < TM THEN T = KO
18250 VTAB 19: HTAB 3: PRINT B$;: HTAB 3: GOSUB 18600
18260 IF F3 = 4 THEN 18550
18290 IF Y < Y1 THEN Y1 = Y
18300 IF Y > Y2 THEN Y2 = Y
18310 X = T / T3 + R1:Y = R2 - Y / Y3
18320 IF F3 = KO AND F = KO AND F5 = K2 THEN HPLLOT XO,YO TO X,Y: GOTO 18550
18330 IF F3 = KO AND (F = 4 OR F5 = K1) THEN HPLLOT X,Y: GOTO 18550
18340 IF F3 = KO THEN 18420
18350 IF F5 = K1 OR F > KO THEN 18550
18359 REM .F5=2,F3=1,F=0
18360 IF SGN (Y - R2) < > SGN (YO - R2) THEN 18550
18370 YN = Y: IF Y < K1 THEN YN = K1: GOTO 18405
18400 IF Y > 158 THEN YN = 158
18405 XN = XO + (X - XO) * (YN - YO) / (Y - YO)
18410 HPLLOT XO,YO TO XN,YN: GOTO 18550
18419 REM .F5=2,F=1,F3=0
18420 IF SGN (Y - R2) < > SGN (YO - R2) THEN 18550
18430 YN = YO: IF YO < K1 THEN YN = K1: GOTO 18470
18460 IF YO > 158 THEN YN = 158
18470 XN = XO + (X - XO) * (YN - YO) / (Y - YO)
18480 HPLLOT XN,YN TO X,Y
18550 F = F3:XO = X:YO = Y
18560 IF PEEK (C7) < C6 THEN NEXT : GOTO 18580
18570 POKE - 16368,KO: GET A$: POKE 54,KO: POKE 55,96: IF A$ = CHR$(5) THEN 185
80
18575 NEXT
18580 V(3) = Y1:V(4) = Y2: POKE 103,P1: POKE 104,P2: RETURN
18598 :
18599 REM ..PRINT 3 SIG FIGS
18600 IF T = KO THEN PRINT T: RETURN
18610 E = INT (K4 * LOG (ABS (T))) - K2
18620 IF E > = KO THEN P = INT (K3 ^ E + K5):Z = INT (ABS (T) / P + K5) * P *
SGN (T): PRINT Z: RETURN
18630 P = INT (K1 / K3 ^ E + K5):Z = INT (ABS (T) * P + K5) / P * SGN (T): PRINT
Z: RETURN
18698 :
18699 REM ..DECODE FUNCTION
18700 CALL 24832
18710 X = FRE (0): PRINT ".....": HTAB 8: GOSUB 20000
18720 IF R3 > K1 THEN PRINT : PRINT "COMMA, SEMI-COLON, & COLON ARE NOT": PRINT "
PERMITTED IN A FUNCTION DEFINITION.": PRINT "REENTER.": HTAB 8: GOTO 18710
18730 A$ = R$(K1):P = 11:Y = KO
18740 IF A$ = " " THEN 18850
18750 IF LEFT$ (A$,K1) = " " THEN A$ = MID$(A$,K2): GOTO 18740
18760 FOR I = K1 TO 22: IF LEFT$ (A$, LEN (R$(I))) = R$(I) THEN 18830
18770 NEXT
18780 Z = ASC (A$):A$ = MID$(A$,K2)
18790 IF Z = ASC ("(") THEN X = X + K1: GOTO 18840
18800 IF Z = ASC (")") THEN X = X - K1: GOTO 18840
18810 IF (Z > ASC ("q") OR Z < ASC (" ")) AND Z < > ASC ("E") AND Z < > ASC
("X") THEN X = 999: GOTO 18860
18820 GOTO 18840
18830 Z = RV(I):A$ = MID$(A$, LEN (R$(I)) + K1): IF Z > 209 AND LEFT$ (A$,K1) <
> "(" AND LEFT$ (A$,K1) < > " " THEN X = 999: GOTO 18860
18840 POKE RI + P,Z:P = P + K1: GOTO 18740
18850 IF P = 11 THEN F = K1: GOTO 18880
18860 POKE RI + P,ASC (""): POKE RI + P + K1,177: POKE RI + P + K2,ASC (""): POKE
RI + P + 3,178
18870 IF X < > KO THEN F = K1: PRINT : PRINT "YOUR FUNCTION IS IMPROPERLY DEFINED
": PRINT "CHECK FOR TYPING OR LOGICAL ERRORS.": CALL - 958: PRINT "Y(X) = ";
R$(K1)
18880 POKE 103,P1: POKE 104,P2: RETURN
19598 :
19599 REM ..CASE CONVERTER

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19600 R$ = "": FOR P = K1 TO LEN (A$):Z = 32
19610 R1$ = MID$(A$,P,1): IF R1$ = "#0" THEN P = P + K1:Z = KO
19620 I = ASC ( MID$(A$,P)): IF I < ASC ("A") OR I > ASC ("Z") THEN Z = KO
19630 R$ = R$ + CHR$( I + Z): NEXT : PRINT R$: RETURN
19996 :
19997 REM ..ARBREADER - GENERAL APPLESOFT INPUT ROUTINE
19998 REM ..PART 1. GET CHARACTERS & ASSEMBLE FIELDS
19999 REM ..SET FIELDS TO NULLS. INITIALIZE FIELD COUNTER
20000 R1$ = R2$(K1): FOR R1 = K1 TO K3:R$(R1) = R1$: NEXT :R3 = K1
20019 REM ..SET ZEROETH CHARCATER TO BLANK
20020 R$ = R2$(K2)
20029 REM ..GET NEXT CHARACTER
20030 R1$ = R$: GET R$
20039 REM ..TEST FOR SPECIAL CHARACTERS
20040 FOR R1 = K1 TO 9
20050 IF R$ = R2$(R1) THEN ON R1 GOTO 20100,20100,20130,20130,20130,20160,20190,2
0240,21180
20060 NEXT
20069 REM ..ADD CHARACTER
20070 R$(R3) = R$(R3) + R$
20079 REM ..TEST FOR CONTROL CHARACTER
20080 IF ASC (R$) < 32 THEN INVERSE :R$ = CHR$( ASC (R$) + 64)
20089 REM ..PRINT CHARACTER
20090 PRINT R$: NORMAL : GOTO 20030
20099 REM ..PROCESS BLANK
20100 IF LEN (R$(R3)) < K1 THEN 20030
20110 IF R1$ = R2$(2) THEN 20030
20120 GOTO 20070
20129 REM ..PROCESS FIELD DELIMITERS
20130 IF LEN (R$(R3)) < K1 THEN 20150
20140 IF R1$ = R2$(2) THEN GOSUB 20300: PRINT R2$(7):
20150 PRINT R$:R3 = R3 + K1: GOTO 20020
20159 REM ..PROCESS CARRIAGE RETURN
20160 IF LEN (R$(R3)) < K1 THEN 20180
20170 IF R1$ = R2$(2) THEN GOSUB 20300
20180 PRINT : GOTO 21000 REM ..THIS IS END OF PART 1
20189 REM ..PROCESS BACK ARROW
20190 IF R3 = K1 AND LEN (R$(R3)) < K1 THEN 20030
20200 IF LEN (R$(R3)) > K1 THEN GOSUB 20300:R$ = RIGHT$( R$(R3),K1): GOTO 20230

20210 IF LEN (R$(R3)) = K1 THEN R$(R3) = R2$(K1): GOTO 20230
20220 IF R3 > K1 THEN R3 = R3 - K1
20230 PRINT R2$(7):R2$(2):R2$(7): GOTO 20030
20239 REM ..PROCESS CONTROL-X
20240 R2 = - K1: FOR R1 = K1 TO R3:R2 = R2 + LEN (R$(R1)) + K1: NEXT
20250 FOR R1 = K1 TO R2: PRINT R2$(7):R2$(K2):R2$(7): NEXT
20260 GOTO 20000
20299 REM ..SHORTEN STRING
20300 R$(R3) = LEFT$( R$(R3), LEN (R$(R3)) - K1)
20310 RETURN
20999 REM ..PART 2. DECODE FOR CATEGORY AND VALUES
21000 FOR R1 = K1 TO R3
21010 R1(R1) = KO
21019 REM ..NULL
21020 IF R$(R1) = R1$(6) THEN R1(R1) = 7:R(R1) = 0: GOTO 21080
21029 REM ..GET VALUE & FIRST CHARACTER
21030 R(R1) = VAL (R$(R1))
21040 R$ = LEFT$( R$(R1),K1)
21049 REM ..NUMERIC
21050 IF R(R1) < > KO OR ASC (R$) = 48 THEN R1(R1) = K1: GOTO 21080
21059 REM ..SPECIAL VALUES
21060 FOR R2 = K1 TO 5: IF R$ = R1$(R2) THEN R1(R1) = R2 + K1: GOTO 21080
21070 NEXT
21080 NEXT
21090 RETURN
21098 :
21099 REM ..ERROR PROCESSING
21100 CALL 26352
21110 R5 = PEEK (222):R6 = PEEK (218) + PEEK (219) * C5
21120 IF R5 = 59 AND R6 = 21030 THEN R1(R1) = 8:R(R1) = KO:R6 = KO: GOTO 21080
21127 IF R6 = 2070 THEN R6 = KO: GOTO 21300
21130 IF R6 = 18010 AND (R5 = 53 OR R5 = 69 OR R5 = 133) THEN 22100
21150 IF R6 = 18230 THEN Y1 = - 1.E37:Y2 = - Y1:F3 = 4:R6 = KO: GOTO 18240
21160 IF R5 < > 255 THEN PR# 0: TEXT : HOME : GOTO 22300
21170 IF F6 = K1 THEN R6 = KO: POKE 54,KO: POKE 55,96: RESUME
21178 :
21179 REM ..CONTROL-C PROCESSING
21180 PRINT "DO YOU WISH TO EXIT (Y/N)?": GET A$: IF A$ = "Y" THEN POKE 103,P1: POKE
104,P2: GOTO 4400

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21200 PRINT "THEN I SHALL CONTINUE.": IF R6 < > KO THEN R6 = KO: GOTO 1000
21210 R1(K1) = KO:R$(K1) = " ": RETURN
21300 IF R5 = 16 OR R5 = 90 OR R5 = 163 OR R5 = 224 THEN 22200
21310 GOTO 2000
22098 :
22099 REM ..FCM EVAL ERROR IN SR18000
22100 PRINT "YOUR FUNCTION CANNOT BE EVALUATED AT": PRINT " X=":T
22110 PRINT "AUTO SCALING NEEDS FCM DEFINED FOR ALL X IN THE INTERVAL. CHANGE TO
MANUAL": PRINT "SCALING OR DIFFERENT FCM DEFINITION."
22130 PRINT " Y(X) = ":Y$
22135 POKE 103,P1: POKE 104,P2
22140 PRINT : PRINT "DO YOU WISH TO CHANGE YOUR FUNCTION": PRINT "(Y/N)?": GET A$
: IF A$ = "Y" THEN 2000
22150 IF A$ = "N" THEN 1000
22160 PRINT CHR$( 7); CHR$( 7): GOTO 22140
22198 :
22199 REM ..MISTAKE IN DEFINING FUNCTION
22200 PRINT : PRINT "YOUR FUNCTION IS IMPROPERLY DEFINED."
22210 PRINT "CHECK FOR TYPING OR LOGICAL ERRORS."
22220 CALL - 958: PRINT "Y(X) = ":Y$:R6 = KO: GOTO 2050
22298 :
22299 REM ..OOPS, AN ERROR WASN'T TRAPPED
22300 POKE 103,P1: POKE 104,P2: PRINT : PRINT "ERROR TYPE ";R5;" OCCURRED IN LINE
"R6:"
22310 PRINT "PROBABLY NOT YOUR FAULT "
22320 PRINT "POWER OFF, THEN ON TO REBOOT."
22330 END
23998 :
23999 REM ..INITIALIZE VARIABLES, SET CONSTANTS
24000 T = 0:X = T:Y = T:DT = .05:TM = .0002
24010 Y1 = T:Y2 = T:T3 = T:Y3 = T:Y4 = T:Z = T:P = T:E = T
24020 DIM V(4),V1(4),Y(200)
24030 KO = 0:K1 = 1:K2 = 2:K3 = 10:K4 = 4343:K5 = 5:K6 = .0001
24040 C1 = 150:C2 = 190:C3 = 84:C4 = 155:C5 = 256:C6 = 128:C7 = - 16384:D$ = CHR$(
4)
24050 GOSUB 2:Y$ = "X*((X-1)^2)^.2"
24059 REM ..VALUES FOR ARBREADER
24060 R1 = K1:R2 = K1:R1$(1) = "Y":R1$(2) = "N":R1$(3) = "H":R1$(4) = "R":R1$(5) =
"R":R1$(6) = "N":R1$(7) = "7"
24070 R2$(1) = "R":R2$(2) = "N":R2$(3) = "N":R2$(4) = "N":R2$(5) = "N":R2$(6) = CHR$(
13):R2$(7) = CHR$( 8):R2$(8) = CHR$( 24):R2$(9) = CHR$( 3)
24080 DIM R$(10),R(10),R1(10),R1$(22),RV(22),LA$(10)
24089 REM ..MACHINE LANG ERROR SR IN ARBPCHAR AT 26352
24090 P1 = "PEEK (103):P2 = PEEK (104)
24110 FOR I = 1 TO 4: READ T$(I): NEXT
24120 DATA "EXMIN=","EXMAX=","YMIN=","YMAX="
24160 PRINT D$:"BLOAD ARBPCHAR"
24180 L1 = P1 + P2 * C5:R1 = PEEK (L1) + PEEK (L1 + K1) * C5
24200 B$ = " ":B1$ = " "
24210 FOR I = K1 TO 13: READ R1$(I):RV(I) = 199 + I: NEXT
24220 FOR I = 14 TO 21: READ R1$(I):RV(I) = 204 + I: NEXT
24230 R1$(22) = "NOT":RV(22) = 198
24240 DATA "+","-","/","AND","OR","<",">","SGN","INT","ABS","SQR","RND","LOG","EXP","COS","SIN","TAN","ATN
24250 F = KO:P1 = K2:F7 = K1:F5 = K2:F8 = K1:F6 = KO
24270 FOR T = K1 TO 10: READ LA$(T): NEXT
24280 DATA "KEEP","CLEAR","AUTOMATIC","MANUAL","SAME","INDEPENDENT","DOTTED","SOL
ID","SHOW " "DON'T SHOW"
24290 V1(K1) = - K1:V1(K2) = K2:T1 = - K1:T2 = K2
24300 RETURN
30123 REM *****
30124 REM ..ARBLOT1, VERSION 2.1 9/4/82
30125 REM ..COPYRIGHT 1982
30126 REM ..BY A R BROWN, JR.
30127 REM 407 PEERY PKWY.
30128 REM GOLDEN, CO 80403
30129 REM .. (303)279-5388

```

Appendix D DISPLAY Listing

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7 REM ..DISPLAY BY A.R.-BROWN, JR.
10 DS = CHR$(4);QS = CHR$(17);CS = CHR$(13);L1 = 25866; POKE - 16368,0: IF PEEK
(24576) = 72 AND PEEK (24580) = 134 AND PEEK (26627) = 8 THEN 63
61 PRINT DS;"BLOADARBPCAR"
63 ONERR GOTO 21400
400 TEXT : HOME : HTAB 11: PRINT "<<DISPLAY SAVER>>": VTAB 6: PRINT "DO YOU WISH T
O": PRINT : PRINT "1. SAVE TO DISK FOR REGULAR SLIDE SHOW.": PRINT : PRINT "2.
SAVE TO DISK FOR COMPRESSED SLIDE SHOW."
130 PRINT : PRINT "3. OUTPUT TO PRINTER.": PRINT : PRINT "4. LOAD IMAGE FROM DISK.
": PRINT : PRINT "5. EDIT DISPLAY.": PRINT : PRINT "6. RETURN TO MAIN MENU.":
150 PRINT : HTAB 20: PRINT "WHICH (1-6)?:": GET AS:F = ASC (AS) - 48: PRINT : IF
F < 1 OR F > 6 THEN HTAB 6: PRINT "ANSWER 1-6": GOTO 150
180 ON F GOTO 1000,4000,3000,2300,7000,1900
190 CALL 1002: PRINT : PRINT DS;"HAPMENU D1"
1000 HOME : HTAB 11: PRINT "<<SLIDE SAVER>>": BS = "SHOW": VTAB 6: PRINT "BUILDING
A SLIDE SHOW REQUIRES AN": PRINT "INITIATED": BS;"DISKETTE"
1020 PRINT : PRINT "DO YOU HAVE A SHOW DISKETTE (Y/N)?:": GET AS: IF LEFT$(AS,1)
= "Y" OR AS = CS THEN 1200
1040 IF LEFT$(AS,1) = "N" THEN 5000
1050 PRINT : PRINT "PLEASE ANSWER YES OR NO.": GOTO 1020
1200 GOSUB 6000: HOME : GOSUB 8000: GOSUB 8010: ON FL GOTO 1210,100
1210 PRINT : PRINT "WHAT NAME DO YOU WISH TO GIVE YOUR": PRINT "DISPLAY": INPUT R
$: PRINT "FILE NAME IS":RS: HTAB 10: PRINT "OKAY(Y/N)?:": GET AS: PRINT : IF
LEFT$(AS,1) < > "Y" AND AS < > CS THEN 1210
1220 PRINT DS;"BSAVE":RS;AS,4000,LS2000,D1: PRINT DS;"LOCK":RS;"D1"
1300 PRINT : PRINT "REMOVE":BS; DISKETTE.": PRINT : PRINT "INSERT ARB PLOT DIS
KETTE.": PRINT : PRINT "AND CLOSE DOOR.": GOSUB 2200: GOTO 100
2200 VTAB 23: PRINT "PRESS RETURN TO CONTINUE.": GET AS: PRINT : RETURN
2300 HOME : HTAB 43: PRINT "<<LOAD IMAGE>>": BS = "IMAGE"
2305 INPUT "FILE NAME?":PS
2310 PRINT : PRINT "SLOT NUMBER?": GET AS:S1 = ASC (AS) - 48: PRINT S1: IF S1 <
1 OR S1 > 7 THEN PRINT "ANSWER 1-7": GOTO 2310
2320 PRINT : POKE - 16368,0: PRINT "DRIVE NUMBER?": GET AS:D1 = ASC (AS) - 48: PRINT
D1: IF D1 < 1 OR D1 > 2 THEN PRINT "ANSWER 1-2": GOTO 2320
2325 PRINT : POKE - 16368,0: PRINT "FILE":PS; "ON DISK IN SLOT":S1;"DRIVE":D
1:"": HTAB 10: PRINT "OKAY(Y/N)?:": GET AS: PRINT : IF LEFT$(AS,1) < > "Y" AND
AS < > CS THEN 2300
2330 PRINT : PRINT "INSERT DISKETTE CONTAINING IMAGE.": GOSUB 2200: PRINT DS;"BLOA
D":PS;AS,4000,VD;"D1";S1:"S":S1: GOSUB 8000: FOR I = 1 TO 600: NEXT I: TEXT : HOME
: VTAB 10: PRINT "IS THIS THE IMAGE (Y/N)?:": GET AS: IF LEFT$(AS,1) < > "Y
" AND AS < > CS THEN 2300
2370 GOTO 1300
3000 TEXT : HOME : HTAB 9: PRINT "<<HARDCOPY DISPLAY>>": VTAB 6: PRINT "YOU CAN MA
KE A PERMANENT COPY OF YOUR": PRINT "DISPLAY ON CERTAIN PRINTERS.": PRINT : PRINT
"DO YOU HAVE": PRINT : HTAB 5: PRINT "1. APPLE SILENTYPE": PRINT : HTAB 5: PRINT
"2. PAPER TIGER W/GRAPHICS OPTION": PRINT : HTAB 5: PRINT "3. MX-80 W/GRAPPLER
": PRINT : HTAB 5: PRINT "4. OTHER"
3060 PRINT : HTAB 20: PRINT "WHICH (1-4)?:": GET AS:F = ASC (AS) - 48: POKE - 16
368,0: IF F < 1 OR F > 4 THEN HTAB 10:"ANSWER 1-4": GOTO 3060
3080 ON F GOTO 3200,3400,3600,3800
4000 HOME : PRINT "<<APPLE SILENTYPE>>": FOR S = 1 TO 7:SA = - 16384 + 256 * S: IF
PEEK (SA + 23) = 201 AND PEEK (SA + 55) = 207 AND PEEK (SA + 76) = 234 THEN
3250
3200 NEXT : PRINT : PRINT "NO SILENTYPE PRINTER INSTALLED!!": GOSUB 2200: GOTO 10
0
3250 GOSUB 8000: GOSUB 8040: ON FL GOTO 3270,100
3270 PRINT : PR# S: PRINT : PR# O: GOSUB 8000: POKE - 12525,64: POKE - 12527,8)
: POKE - 12529,255: POKE - 12528,7: POKE - 12524,0: PR# S: PRINT QS;CHR$(S
): PRINT : PR# O: IN# O: GOTO 400
3400 HOME : HTAB 11: PRINT "<<PAPER TIGER>>": PRINT : PRINT "PAPER TIGER MUST HAVE
GRAPHICS OPTION": PRINT "AND STANDARD APPLE SERIAL OR PARALLEL.": PRINT "INTER
FACE.": PRINT : HTAB 5: PRINT "1. SERIAL": PRINT : HTAB 5: PRINT "2. PARALLEL
": PRINT : HTAB 5: PRINT "3. NON-STANDARD"
3430 PRINT : HTAB 10: PRINT "WHICH (1-3)?:": GET AS:F1 = ASC (AS) - 48: IF F1 < 1
OR F1 > 3 THEN HTAB 10: PRINT "ANSWER MUST BE 1-3.": GOTO 3430
3452 IF F1 = 3 THEN 100
3455 PRINT : PRINT "SLOT CONTROLLER # (1-7)?:": GET AS:S = ASC (AS) - 48: IF S <
1 OR S > 7 THEN PRINT "SLOT# MUST BE BETWEEN 1 AND 7.": GOTO 3455
3470 POKE L1 + 2,32: POKE L1 + 1,7: POKE L1 + 4,72: POKE L1 + 2,169: POKE L1 + 3,0
3480 POKE L1 + 4,141: POKE L1 + 5,248 + S: POKE L1 + 6,S: POKE L1 + 7,104: POKE L
1 + 8,96: GOTO 3500
3490 POKE L1 + 8,128 + S * 16
3500 POKE 25856,64: GOSUB 8000: GOSUB 8040: ON FL GOTO 3510,100
3510 GOSUB 8000: PRINT DS;"PR#":S: PRINT : PRINT DS;"PR#O": CALL 25879: GOTO 100
3520 HOME : PRINT "<<GRAPHER/MISO>>"
3610 PRINT : PRINT "CONTROLLER SLOT#(1-7)?:": GET AS:S = ASC (AS) - 48: IF S < 1 OR
S > 7 THEN PRINT "SLOT MUST BE BETWEEN 1&7.": GOTO 3610
3620 PRINT : PRINT "SINGLE OR DOUBLE SIZE GRAPH(S/D)?:": GET SS: PRINT : IF SS < >

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"s" AND SS < > "D" THEN 3620
3630 GOSUB 8000: GOSUB 8040: ON FL GOTO 3640,100
3640 GOSUB 8000: PR# 8: IF SS = "S" THEN PRINT CHR$ (9);"G2R": PR# 0: GOTO 400
3650 PRINT CHR$ (9);"G2R": PR# 0: GOTO 100
3800 HOME : VTAB 10: PRINT "FOR OTHER PRINTERS, SEE INSTRUCTIONS IN ARBLOT MANUAL
, SECTION 2.5": GOSUB 2200: GOTO 100
4000 HOME : HTAB 4: PRINT "<<HARRIS COMPRESSED SLIDE SAVER>>";BS = "GRAPH PACK": VTAB
6: PRINT "BUILDING A COMPRESSED SLIDESHOW": PRINT "REQUIRES AN INITIATED";BS:
PRINT "DISKETTE,"
4020 PRINT : PRINT "DO YOU HAVE A ";BS;" DISKETTE(Y/N)?";: GET AS: IF LEFT$ (AS,1
) = "Y" OR AS = CS THEN 4200
4040 IF LEFT$ (AS,4) = "N" THEN 5000
4050 PRINT "PLEASE ANSWER YES OR NO": GOTO 4020
4200 GOSUB 6000: PRINT DS;"RUN";BS;"D1"
5000 HOME : PRINT "TO PREPARE YOUR SLIDESHOW DISKETTE ": PRINT "YOU MUST DO THE FO
LLOWING": PRINT : PRINT "1. REMOVE THE ARBLOT DISKETTE": PRINT "2. INSERT B
LANK DISKETTE & CLOSE DOOR": PRINT "3. TYPE 'INI';BS;":
5040 PRINT "4. WAIT 30 SECONDS UNTIL LIGHT ON": PRINT "5. DISK DRIVE GOES OUT & CU
RSOR APPEARS.": PRINT "6. REMOVE ";BS;" DISKETTE": PRINT "7. INSERT ARBLOT
DISKETTE & CLOSE DOOR.": PRINT "8. TYPE 'RUN DISPLAY' TO GET BACK TO": PRINT
SAVING YOUR DISPLAY.":
5080 PRINT : HTAB 10: PRINT "START WHEN THE CURSOR REAPPEARS.": PRINT DS;"LOAD";BS
;"D1"
6000 HOME : PRINT "REMOVE ARBLOT DISKETTE ": PRINT : PRINT " INSERT ";BS;" DISK
ETTE.": PRINT : PRINT " AND CLOSE DOOR": PRINT : PRINT "PRESS RETUR
N WHEN READY TO CONTINUE.": GET AS: PRINT : RETURN
7000 HOME : HTAB 10: PRINT "<OH GRAPH EDITOR>": PRINT : PRINT DS;"RUNNH GRAPH ED
ITOR.D1"
8000 POKE - 16304,0: POKE - 16302,0: POKE - 16297,0: POKE - 16299,0: RETURN
8010 FOR I = 1 TO 600: NEXT I: TEXT : HOME : VTAB 10: PRINT "IS THIS THE IMAGE (Y/
N)?": GET AS: IF AS = "Y" THEN FL = 1: RETURN
8020 IF AS = "N" THEN FL = 2: RETURN
8030 PRINT CHR$ (7); CHR$ (7): GOTO 8010
21100 CALL 26352:R5 = PEEK (222):R6 = PEEK (218) + PEEK (219) * 256: IF R5 = 4 AND
R6 = 1220 THEN PRINT "DISKETTE IS WRITE PROTECTED.": PRINT "YOU MUST INSERT S
HOW DISKETTE.": GOTO 1020
21430 IF R5 = 9 AND R6 = 1220 THEN PRINT "DISKETTE IS FULL.": PRINT "YOU MUST STA
RT A NEW DISKETTE.": PRINT DS;"DELETE";R5: GOTO 1020
21140 IF R5 = 10 AND R6 = 1220 THEN PRINT "YOU HAVE ALREADY USED THAT NAME.": PRINT
"PICK ANOTHER.": GOTO 1210
21145 IF R5 = 6 AND R6 = 2330 THEN PRINT "FILE NOT FOUND": GOTO 2305
21150 IF R5 = 255 THEN TEXT : GOTO 190
21170 PRINT "ERROR TYPE ";R5;" IN LINE ";R6: PRINT "PROBABLY NOT YOUR FAULT.": PRINT
"TRY 'RUN DISPLAY' TO RECOVER.": PRINT "IF UNSUCCESSFUL, POWER OFF, THEN ON."

```

Appendix E Error Reporting

ARBPLOT is intended to be error-free, easy to understand, and straightforward to use. However, it is a truism in computing that the only bug-free program is one which is ready to be discarded. Therefore, the programs have a general message which we hope you will never see.

ERROR TYPE NN OCCURRED IN LINE XXX.
PROBABLY NOT YOUR FAULT.
POWER OFF, THEN ON TO REBOOT.

NN is the error code on page 136 of the Applesoft II Reference Manual or page 114 of the DOS Manual. XXX is the statement number in the program being run.

Here are the errors you are most likely to encounter.

CODE	ERROR TYPE	PROBABLE CAUSE
0	NEXT without FOR	Program bug; tell us.
1	Language not available	Applesoft missing or inoperative on your computer. See your local expert.
4	Write protected	Write-protect tab on diskette; are you trying to write an ARBPLOT disk?
6	File not found	File not on diskette; do you have wrong name or wrong diskette?
8	I/O error	Is door open?
9	Disk full	SHOW or GRAPH PACK diskette full.
11,16	Syntax error	Program bug; tell us.
22-224	Various errors	Program bug; tell us.
Others		Mystery; tell us.

If you encounter a problem -- whether you receive the message above, the program self-destructs (this has happened in development), or something is simply awkward -- we would like to hear from you.

Write to:

CONDUIT
P.O. Box 388
Iowa City, Iowa 52244

Include:

1. Program name.
2. All program menu settings.
3. If possible, program settings and actions just prior to the run that bombed.
4. Results: error message or other symptoms.

When the problem has been corrected, you will receive an update.

Appendix F

ARBREADER

A General-Purpose Input Subroutine for Applesoft II Programs

ARBREADER is a general-purpose input routine with the following properties:

1. An input record consists of any string of characters terminated by a carriage return. Any character can be included in a field except comma, semicolon, colon, carriage return, back arrow, Ctrl-X, or Ctrl-C, as specified in line 24070.
2. An input record is divided into fields, with fields separated by commas, semicolons, or colons. Field delimiters can be changed by changing the contents of R2\$(3), R2\$(4), or R2\$(5) in line 24070.
3. Leading or trailing blanks or redundant imbedded blanks are rejected. If you try to enter extra blanks, the cursor will not move but will await entry of your next non-blank character.
4. Control characters except Ctrl-C, Ctrl-H, Ctrl-X and Ctrl-@ are displayed as the corresponding inverse character. However, they are stored as the appropriate control character.
5. Characters within a record can be deleted by use of the back-arrow or Ctrl-H. The character is immediately removed both from the record and the display.
6. The entire record can be deleted by Ctrl-X. The record is immediately removed both from memory and the display, leaving your program awaiting input of a new record.

7. All control functions except "RESET" are bypassed. In particular, the editing characters and Ctrl-C are disabled. "RESET" still returns you to the monitor. Ctrl-C is trapped so that it is under the control of your program.
8. Ctrl-@ is interpreted as the null character, which is ignored unless it is the only character in a field.
9. The double quote (") may be entered just as any other character.

In addition, ARBREADER provides a vehicle by which you can have complete control over errors occurring in execution of your program. Insert your own error handling routine beginning in line 21125 (lines 21100-21120 are needed for ARBREADER error control).

Enter ARBREADER by "GOSUB 20000". No parameters are passed to ARBREADER. One variable and three arrays are returned:

- R3 = the number of fields in the record
- R\$(R3) = the array of contents of input fields in string form
- R(R3) = the array of numerical values of field contents. If the beginning of a field can be interpreted as a number, its value is returned. If either the beginning of the field cannot be interpreted as a number or the numerical value is too large (overflow), the value returned is zero.
- R1(R3) = the array of coded responses

Values of coded field contents are:

- 0 = Regular string
- 1 = Numeric
- 2 = Y(es)
- 3 = N(o)
- 4 = H(elp) or ?
- 5 = R(eenter)
- 6 = E(xit) or Ctrl-C
- 7 = Null
- 8 = Overflow

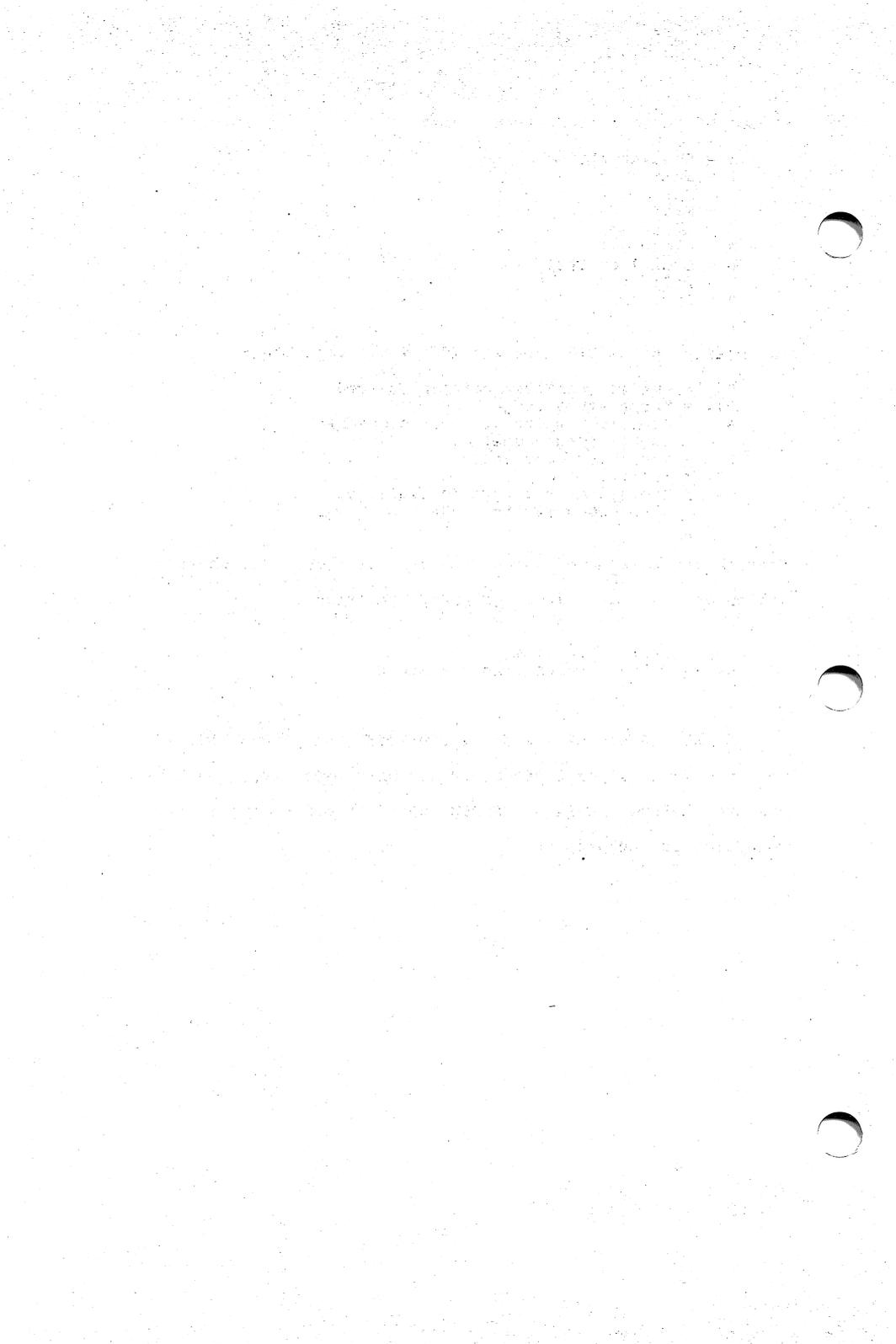
Internally, ARBREADER uses six additional variables:

- R\$ = Current character being processed
- R1\$ = Temporary storage
- R = Numerical value of field contents
(zero if non-numeric)
- R1 = Temporary code value
- R2 = Index
- R4 = Dimension of arrays (default=10,
changeable in line 24080)

When an error is encountered, two variables are set, whose values are then available for error processing.

- R5 = error type code
- R6 = line in which error occurred

ARBREADER occupies lines 24060-24080 and 20000-21120 of your program. Lines 24060-24080 and ONERR GOTO 21100 must be executed before the first "GOSUB 20000" or before any error condition is encountered.



Appendix G Summary of Actions

In general, respond to any menu, yes/no, or other inherently single character question with just the character (no "RETURN"). Respond to any request for numerical data with the value followed by "RETURN"; "RETURN" alone confirms the previous value.

a. **FUNCTION.** Enter a valid Applesoft BASIC arithmetic expression using only the variable name appearing on the left side of the equation (see Section 2.4). ARBPLOT4 will also accept INVERSE. CONIC takes coefficients of a quadratic equation or its polar equivalent. (Same rules for derivatives and antiderivatives.)

b. **LIMITS/SCALING.** Limits are numeric except for "C" in ARBSEQ and ARBINT1, and "D" in ARBINT1. AUTOMATIC scaling fills the screen; MANUAL scaling uses limits you specify, and the curve may go offscale. You must use MANUAL scaling if there are gaps in the domain.

c. **SCREEN: KEEP or CLEAR PRESENT DISPLAY.** With CLEAR, the entire screen is cleared before plotting. With KEEP, the new curve is overlaid on previous ones; data and equations shown refer only to the latest curve.

d. **CURVE: SOLID or DOTTED LINE.** This command has no effect on anything else; it simply helps distinguish multiple

curves. In ARBPLOT3, a solid curve means R is positive, while a dotted curve means R is negative. In ARBSEQ, the sequence is solid, sum is dotted, and function is dashed.

e. AXES: SHOW or DON'T SHOW. This command has no effect on anything else. The axes are shown or not.

f. MODE, METHOD, or PROBLEM. These commands are unique to each program. See Chapter 3.

g. SHOW CURRENT DISPLAY. This command shows the most recently created display, regardless of where it came from.

h. PLOT CURVE or special action. In some programs, you may interrupt the plotting process by pressing any key. If you then press any key except Ctrl-E, the program will resume the plotting or other specified process. Ctrl-E after interrupt will cause you to exit from plotting; with some programs exit is immediate, with others you must press a key after "End". In any case, press a key after "End" to exit. ARBLIM, ARBROOT, and CONIC require "E" to exit; other keys take action on the screen.

i. EXIT. Press "8" confirmed by "Y" to return to the main ARBPLOT menu. EXIT at the main ARBPLOT menu then lets you go to the other ARBPLOT diskette or exit to Applesoft.

Appendix H

Making Backup Copies of ARBPLOT

ARBPLOT can be copied by any standard copy program, e.g., COPYA from the Apple II System Master diskette. We recommend making copies and storing the original in a safe place. However, in order for programs on the copy to load faster than the normal speed of the original, a few extra steps must be added to the copying process. Here is the procedure:

1. Boot the System Master diskette.
2. Insert ARBPLOT Disk 2 into your drive and type
BRUN MH SECTOR SKEW.
3. Insert the System Master diskette and type RUN COPYA.
4. Follow the directions given by the COPYA program to copy both ARBPLOT disks.

You now have a copy that will run faster than the original. After you have finished copying, you must:

5. Boot any disk with the normal DOS on it (such as the System Master or either ARBPLOT disk). This is necessary because the program MH SECTOR SKEW altered the DOS originally in memory just for copying.

This procedure will speed up file-loading by about 65% on any standard diskette you copy. Time spent saving files will

be reduced as well unless the file is being saved under the name of another file already on the disk. In this special case (which occurs often in program development), saving a file actually takes about 33% longer than normal. For this reason, we recommend that you use this method only for copying diskettes on which no further saving will be done.

Rest assured that your "improved" copies contain no bizarre alterations to DOS on them. If your copy is in turn copied by a standard copy program (e.g, COPYA without using MH SECTOR SKEW), the new diskette will resume the slower loading characteristics of the original. The trick to the speed-up is "nine-descending sector skewing," described in D. Worth and P. Lechner, "The Speed Sector," Softalk, April 1982.



